

Semileptonic decays of D mesons in unquenched lattice QCD*

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We present our preliminary results for semileptonic form factors of D mesons in unquenched lattice QCD. Simulations are carried out with $n_f = 2 + 1$ dynamical quarks using gauge configurations generated by the MILC collaboration. For the valence quarks, we adopt an improved staggered light quark action and the clover heavy quark action. Our results for $D \rightarrow K$ and $D \rightarrow \pi$ form factors at $q^2 = 0$ are in agreement with the experimental values.

1. INTRODUCTION

In order to extract the CKM matrix element $|V_{ub}|$ accurately with experimental measurement of the semileptonic decay width, a precision lattice QCD calculation of the $B \rightarrow \pi$ form factor is required. To check the reliability of lattice calculations of the heavy to light form factors, study of the semileptonic decays of D mesons, such as $D \rightarrow K$ and $D \rightarrow \pi$, is a good test ground because the corresponding CKM matrices $|V_{cs}|$ and $|V_{cd}|$ are known more accurately than $|V_{ub}|$. Furthermore, forthcoming experiments by the CLEO-c collaboration will provide more stringent checks of lattice calculations in the D meson system.

We have started new lattice calculations of heavy quark physics in unquenched QCD[1], and here we report our preliminary results for semileptonic form factors of D mesons. We are using unquenched gauge configurations with $n_f = 2 + 1$ improved staggered quarks generated by the MILC collaboration[2], with which the systematic errors due to the quenched approximation should be almost absent. For the valence light quarks, we adopt an improved staggered quark action,

which allow us to simulate at lighter quark mass than previous studies with the Wilson-type light quarks. Hence, our new calculations should have a better control over the chiral extrapolations.

2. METHOD

In order to combine the staggered light quark with the Wilson-type heavy quark in heavy-light bilinears, we convert the staggered quark propagator $g(x, y)$ to the “naive” quark propagator $G(x, y)$ according to

$$g(x, y) \Omega(x) \Omega^\dagger(y) = G(x, y) \quad (1)$$

with $\Omega(x) = \gamma_0^{x_0} \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3}$ [3]. The 3-point function for the matrix element is then computed as

$$C_{3,\mu}^{D \rightarrow \pi}(t_x, t_y; \mathbf{p}_\pi, \mathbf{p}_D) = \sum_{\mathbf{x}, \mathbf{y}} e^{i(\mathbf{p}_D - \mathbf{p}_\pi)\mathbf{y} - i\mathbf{p}_D\mathbf{x}} \times$$

$$\langle \text{Tr}[g_d^\dagger(y, 0) \Omega^\dagger(y) \gamma_5 \gamma_\mu G_c(y, x) \gamma_5 \Omega(x) g_u(x, 0)] \rangle,$$

where subscripts d, c, u denote quark flavors. The matrix element can be extracted from the ratio

$$\langle \pi | V_\mu | D \rangle \stackrel{t_x \gg t_y \gg 0}{\sim} \frac{C_{3,\mu}^{D \rightarrow \pi}(t_x, t_y; \mathbf{p}_\pi, \mathbf{p}_D)}{C_2^\pi(t_y, \mathbf{p}_\pi) C_2^D(t_x - t_y, \mathbf{p}_D)} \quad (2)$$

with the D meson (Wilson-Naive) 2-point function C_2^D and the pion (Naive-Naive) 2-point function C_2^π . Care is needed, however, for the overall

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Table 1

Quark mass, statistics and the sink time.

$m_l^{\text{sea}}/m_s^{\text{sea}}$	$m_l^{\text{val}}/m_s^{\text{val}}$	conf	t_x
0.01/0.05	0.01/0.0415	552×4	20
0.02/0.05	0.02/0.0415	460	20
0.03/0.05	0.03/0.0415	358	22
0.01/0.05	0.0415/0.0415	412	26
∞/∞	0.0415/0.0415	350	16

normalization of amplitude since the naive quark action describes 16 fermions, which can cause the doubling of these correlation functions.

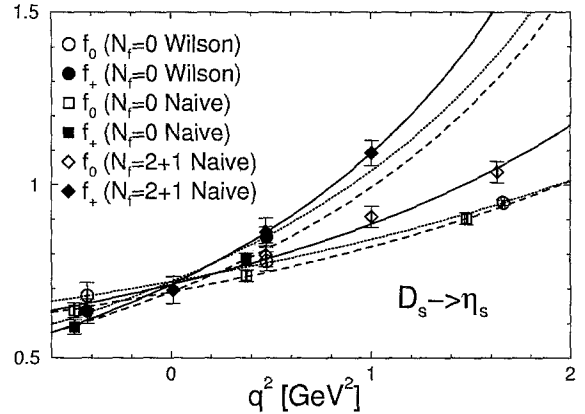
In Ref. [3] it is shown that the Wilson-Naive 2-point function C_2^D does not have the doubling because contributions of quarks with momentum $p \sim \mathcal{O}(\pi/a)$ are suppressed by the Wilson term. The same also holds for the 3-point functions which include at least one Wilson propagator such as $C_{3,\mu}^{D \rightarrow \pi}$. On the other hand, the Naive-Naive 2-point function C_2^π should have 16 equivalent contributions. Therefore one has to divide it by 16 to get the physical amplitude; $C_2^{\pi, \text{phys}} = C_2^\pi/16$.

3. SIMULATION

Unquenched calculations are performed using $n_f = 2 + 1$ dynamical gauge configurations obtained with an improved staggered quark action on a $20^3 \times 64$ lattice ($a^{-1} \approx 1.58$ GeV)[2]. For the valence light quarks we use the same staggered quark action as for the dynamical quarks. The valence light quark (u, d) mass m_l^{val} is usually set equal to the dynamical light quark mass m_l^{sea} . For the valence charm quark we use the clover action with the Fermilab interpretation[4]. The hopping parameter is fixed to $K_{\text{charm}} = 0.119$, based on our spectrum study[1]. The $\mathcal{O}(a)$ rotation[4] is performed for the vector current.

The 3-point functions are computed in the D meson rest frame ($\mathbf{p}_D = \mathbf{0}$) for the light meson momentum \mathbf{p}_π up to $(1, 1, 1)$ in lattice units, using local source and sink. The sink time is fixed to $t_x = 20 - 26$ depending on m_l^{val} , whereas the source time is set to $t_0 = 0$ with an exception at $m_l^{\text{val}} = 0.01$, where we average over results from four source times $t_0 = 0, 16, 32$ and 48 . Some simulation parameters are summarized in Table 1.

In addition to the unquenched calculations, we

Figure 1. $D_s \rightarrow \eta_s(s\bar{s})$ form factors.

also perform a quenched simulation at $m_l^{\text{val}} = m_s^{\text{val}} = 0.0415$ using $\beta = 5.9$ ($a^{-1} \approx 1.80$ GeV) configurations on a $16^3 \times 32$ lattice used in our previous study[5]. Comparison between the quenched result with the staggered light quarks and that with the Wilson-type light quarks[5] allows us to check the validity of our new calculations.

For the vector current renormalization $Z_{V_\mu}^{cd}$ we follow the method in Ref. [5]. We take $Z_{V_\mu}^{cd} = \rho_{V_\mu} (Z_V^{cc} Z_V^{dd})^{1/2}$, where Z_V^{qq} ($q = c, d$) is the renormalization constant for the flavor-conserving current, which we compute nonperturbatively from the charge normalization condition $Z_V^{qq} \langle D(\mathbf{0}) | V_4^{qq} | D(\mathbf{0}) \rangle = 2m_D$. The ρ_{V_μ} is set to unity. The one-loop calculation is in progress.

4. RESULTS

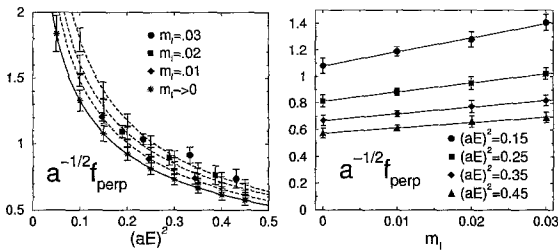
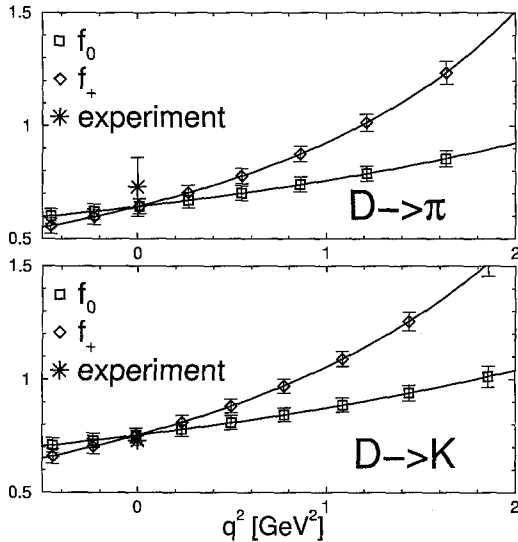
Form factors are defined through

$$\begin{aligned} \langle \pi | V^\mu | D \rangle &= f_+(q^2) \left[p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2} q \right]^\mu \\ &\quad + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \\ &= \sqrt{2m_D} \left[v^\mu f_\parallel(E) + p_\perp^\mu f_\perp(E) \right] \end{aligned}$$

with $q = p_D - p_\pi$, $v = p_D/m_D$, $p_\perp = p_\pi - Ev$ and $E = E_\pi$. The second expression using f_\parallel and f_\perp is more convenient when one considers the heavy quark expansion and the chiral limit.

4.1. $D_s \rightarrow \eta_s$

In Fig. 1 we summarize the results of form factors f_0 and f_+ for the $D_s \rightarrow \eta_s(s\bar{s})$ decay

Figure 2. Chiral extrapolation for $f_{\perp}^{D \rightarrow \pi}$.Figure 3. $D \rightarrow \pi$ and $D \rightarrow K$ form factors.

obtained with the naive(staggered) light quarks and previous results[5] with the Wilson-type light quarks. The lines in the figure are fitting curves with a parametrization by Becirevic and Kaidalov (BK)[6]. One can see a nice agreement between the quenched result with the naive quarks (squares, dashed line) and that with the Wilson-type quarks (circles, dotted), showing that our new method works well. We also note that the unquenched result (diamonds, solid) is larger than quenched ones for $q^2 > 0$. See also Ref. [7] for a similar comparison for the $B_s \rightarrow \eta_s$ form factors.

4.2. $D \rightarrow \pi$ and $D \rightarrow K$

To obtain $D \rightarrow \pi/K$ form factors at the physical quark mass, we need to perform a chiral extrapolation using data in range of $m_l^{\text{val}} = 0.01-0.03$. We do this for f_{\parallel} and f_{\perp} at fixed pion(kaon) energies $E_{\pi(K)}$ because the chiral perturbation

formulas for the heavy to light form factors are given in such a way[8]. In order to interpolate and extrapolate the results to common values of E_{π} , we use a fit with the BK parametrization. We then perform a linear chiral extrapolation in m_l^{val} at nine values of $(aE_{\pi})^2$. One example of these procedures is shown in Fig. 2 for $f_{\perp}^{D \rightarrow \pi}$. Finally f_{\parallel} and f_{\perp} are converted to f_0 and f_+ .

The $D \rightarrow \pi$ and $D \rightarrow K$ form factors are shown in Fig. 3 together with experimental values at $q^2 = 0$ [9]. Our results at $q^2 = 0$ are

$$f_+^{D \rightarrow K}(0) = 0.75(3), \quad f_+^{D \rightarrow \pi}(0) = 0.64(3) \quad (3)$$

with statistical errors only, whereas experimental values are $f_+^{D \rightarrow K}(0) = 0.73(2)$ and $f_+^{D \rightarrow \pi}(0) = 0.73(13)$ with $|V_{cs}| = 0.996(13)$ and $|V_{cd}| = 0.224(16)$ [9]. Our results are in agreement with the experimental values. The analysis including the chiral logarithm and the one-loop renormalization constant is underway.

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REFERENCES

1. See also, P. Mackenzie and J. Simone, these proceedings.
2. C. Bernard *et al.* (MILC collaboration), Phys. Rev. D **64**, 054506 (2001).
3. M. Wingate *et al.* Phys. Rev. D **67**, 054505 (2003)
4. A. X. El-Khadra, A. Kronfeld and P. Mackenzie, Phys. Rev. D **55**, 3933 (1997).
5. A. X. El-Khadra *et al.* Phys. Rev. D **64**, 014502 (2001)
6. D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478**, 417 (2000)
7. C. DeTar, these proceedings.
8. D. Becirevic, S. Prelovsek and J. Zupan, Phys. Rev. D **67**, 054010 (2003)
9. K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002); C. Caso *et al.* Eur. Phys. J. C **3**, 1 (1998).