

Properties of charmonium in lattice QCD with $2 + 1$ flavors of improved staggered sea quarks

Massimo di Pierro^a, Aida X. El-Khadra^b, Steven Gottlieb^c, Andreas S. Kronfeld^d, Paul B. Mackenzie^d, Damian P. Menscher^b, Mehmet B. Oktay^b, Masataka Okamoto^d, and James N. Simone^{d*}

^aSchool of Computer Science, Telecommunications, and Information Systems, DePaul University, Chicago, IL 60604

^b Department of Physics, University of Illinois, Urbana, IL 61801

^c Department of Physics, Indiana University, Bloomington, IN 47405

^d Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510

We use the dynamical gluon configurations provided by the MILC collaboration in a study of the charmonium spectrum and ψ leptonic width. We examine sea quark effects on mass splitting and on the leptonic decay matrix element for light masses as low as $m_l/5$, while keeping the strange quark mass fixed and the lattice spacing nearly constant.

1. INTRODUCTION

Charmonium states below open flavor threshold are considered gold-plated quantities in lattice QCD. They are almost stable mesons, which should be accurately calculable in lattice QCD once realistic sea-quark effects are included in the simulations. A high precision study of the charmonium system in unquenched lattice QCD is interesting for several reasons. First, it provides us with an important test of the lattice methods, because the methods used for charmonium calculations are similar to those used for CKM determinations in the D meson system [1,2]. Second, it allows us to test our improved actions which are under development [3], since different splittings in the charmonium system are sensitive to different correction operators in the action. For example, the hyperfine splitting is very sensitive to $\bar{\psi}\sigma \cdot B\psi$, while the χ_c fine structure is expected to sensitively depend on $\bar{\psi}\sigma \cdot (D \times E)\psi$. Finally, together with 2-loop perturbation theory, it yields precise determinations of α_s and the charm quark mass.

The $2 + 1$ dynamical gauge configurations gen-

erated by the MILC collaboration contain realistic sea quark effects, since they reach the chiral region ($m_l \geq m_s/5$) – a necessary feature to control chiral extrapolation errors. A first test of lattice QCD calculations which use the MILC configurations was presented in Ref. [9]. For the first time agreement (at the few % level) with experiment was achieved for a variety of different physical systems, involving b , c , and light quarks. This comparison includes our results for the $1P-1S$ splitting in charmonium which are also presented here.

The work presented here continues our charmonium study [4,5]. Our companion study of the D_s and D meson spectra and weak decays is presented in Ref. [1,2].

2. METHODS

We are using the MILC collaboration “Asqtad” gluon ensembles [6,7]. The Asqtad action has leading $\mathcal{O}(\alpha_s^2 a^2)$ gluon uncertainties and leading $\mathcal{O}(\alpha_s a^2)$ uncertainties for the improved staggered sea quarks.

The gluon ensembles have one flavor approximating the strange quark and two equal-mass

*Talk presented by J. Simone, simone@fnal.gov

lighter flavors. A matched set of gluon ensembles is available having light masses in the range m_s to $m_s/5$ and nearly constant lattice spacing (see Table 1).

Our charm quarks are $\mathcal{O}(a)$ -improved Wilson fermions in the Fermilab interpretation of heavy quarks. The coefficient of the clover term has the tadpole-improved tree-level value. The bare charm quark mass is tuned by demanding that the D_s meson kinetic mass equal the experimental value.

3. CHARMONIUM SPECTRUM

Fig. 1 shows the overall picture of the charmonium spectrum after sea quark extrapolations. The zero of energy is taken to be the spin average of the $1S$ masses. The lattice spacing is determined using the $h_c(1P)$ splitting as input. This lattice spacing is consistent, at the few percent level, with other ways of setting the lattice spacing [9]. We compare the charmonium $1P$ - $1S$ and bottomonium $2S$ - $1S$ lattice spacings in Table 1.

Table 1

The bare light sea quark mass and coupling β for the MILC three-flavor gluon ensembles. The strange quark has mass $am_s = 0.05$. Lattice spacings are given in GeV. The $2S$ - $1S$ results are from Reference [8].

am_l	beta	cfgs	$a_\psi^{-1}(1P-1S)$	$a_\Upsilon^{-1}(2S-1S)$
.007	6.76	403	1.55(3)	—
0.01	6.76	593	1.56(2)	1.59(2)
0.02	6.79	460	1.59(3)	1.61(2)
0.03	6.81	549	1.57(3)	1.60(3)

We extrapolate linearly in the light sea quark mass. The mass dependence is mild: linear terms are typically of the same order of magnitude as their statistical error. The mass dependence for the hyperfine splitting, shown in Fig. 2, illustrates a typical extrapolation. Smaller statistical errors and more sea quark masses would be needed to better resolve terms in the chiral expansions.

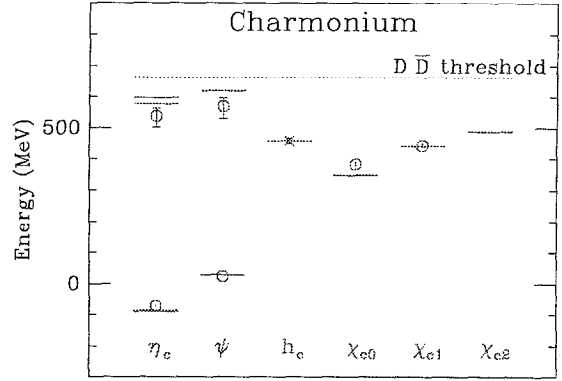


Figure 1. The charmonium spectrum in $2+1$ flavor lattice QCD after extrapolation to the physical sea quark masses. The lattice spacing is $a \approx 1/8$ fm. No lattice result for the $\chi_{c2}(1P)$ splitting is shown since that state was not computed in this study. The dotted indicates the $D\bar{D}$ threshold energy.

The $2S$ splittings shown in Fig. 1 have large errors. Statistical uncertainties for these excited states are 20 – 30 times as large as uncertainties in the ground states with the same J^{PC} . Ground state and radially excited state energies are obtained from a single fit using Bayesian techniques. We continue to investigate ways to improve the signal for excited states.

3.1. Hyperfine splitting

The hyperfine splitting in charmonium is a gold-plated quantity, which must agree with experiment once all known systematic errors are corrected.

As shown in Fig. 2 we obtain a $1S$ hyperfine splitting of 97 ± 2 MeV, or in ratio to experiment, theory/expt = 0.82 ± 0.02 . A comparison to our previous quenched result (obtained at similar lattice spacings), theory/expt = 0.6, shows that sea quarks have an appreciable effect on this quantity. The remaining discrepancy is likely due to having only a (tadpole improved) tree-level estimate for the coefficient of the $\sigma \cdot B$ term in our action. A one-loop calculation of this coefficient is in progress [10].

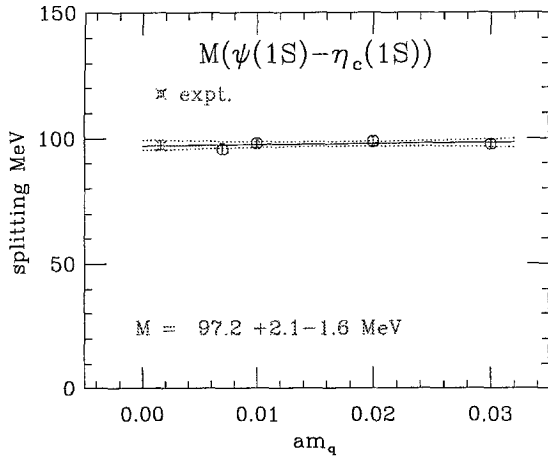


Figure 2. The charmonium 1S hyperfine splitting as a function of the mass of the two light sea quarks. The solid and dotted lines show the linear extrapolation with 68% confidence bounds. The extrapolated value is denoted by the burst symbol. The experimental value lies above the lattice determination.

4. $\psi(1S)$ LEPTONIC WIDTH

We determine the hadronic matrix element, $V_\psi \equiv \langle 0 | V_j | \psi \rangle$ for the leptonic width from the overlap coefficient of the two-point function of the ψ propagator annihilated by the local vector current. At present our calculation does not include the $O(a)$ correction for the vector current.

For the spatial current renormalization, we use the formula $Z_{V_i} = \rho_{V_i} Z_{V_4}$, and the nonperturbative result for Z_{V_4} from Ref. [2]. Since the one-loop correction to ρ_{V_i} is currently unknown, we have $\rho_{V_i} = 1$ in our calculation of V_ψ . Hence, our results for this quantity are very preliminary.

The experimental measurement of the leptonic width implies for the matrix element $V_\psi^{\text{expt}} = 0.504 \pm 0.018 \text{ GeV}^{3/2}$.

Fig. 3 shows our preliminary results for V_ψ ; after sea quark extrapolation we find $V_\psi^{\text{thy}} = 0.586 \pm 0.015 \text{ GeV}^{3/2}$. The statistics only uncertainty is dominantly from the lattice spacing determination. We find theory/expt = 1.16 ± 0.04 , combining errors in quadrature.

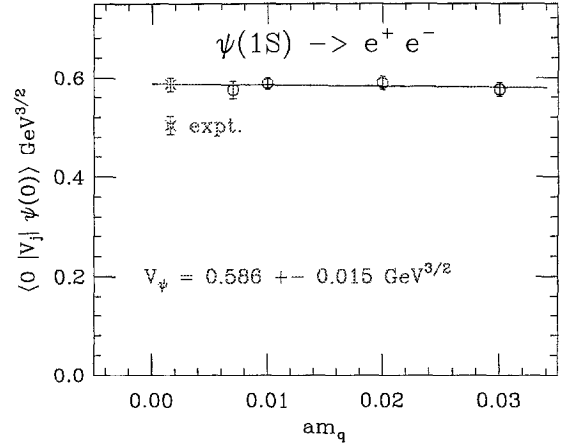


Figure 3. The annihilation matrix element for the decay $\psi(1S) \rightarrow e^+ e^-$ as a function of the mass of the two light sea quarks. The extrapolated matrix element lies above the value derived from the experimental leptonic decay width.

ACKNOWLEDGMENTS

We thank the MILC collaboration for their gauge configurations. These calculations were done on the PC clusters deployed at Fermilab under the DoE SciDAC program. This work was supported in part by the DoE.

REFERENCES

1. P. Mackenzie, these proceedings.
2. M. Okamoto, these proceedings.
3. M. Oktay, these proceedings.
4. A. X. El-Khadra *et al.*, Nucl. Phys. Proc. Suppl. **83**, 283 (2000).
5. M. Di Pierro *et al.*, [arXiv:hep-lat/0210051].
6. Steven Gottlieb, these proceedings.
7. C. W. Bernard *et al.*, Phys. Rev. D **64**, 054506 (2001) [arXiv:hep-lat/0104002].
8. M. Wingate, these proceedings.
9. HPQCD Collaboration: C. T. Davies *et al.*, [arXiv:hep-lat/0304004].
10. M. Nobes and H. Trotter, these proceedings.