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The Second Moment of the Pion Light Cone Wave Function

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We present a preliminary result for second moment of the light cone wave function of the pion. This parameter is the subject of a discrepancy between theoretical predictions (coming from lattice and sum rules) and a recent experimental result (that remarkably agrees with purely perturbative predictions). In this work we exploit lattice hypercubic symmetries to remove power divergences and, moreover, implement a full l-loop matching for all the contributing operators.

1. INTRODUCTION

The light cone wave function of the pion, $\phi(x,Q^2)$ is the probability amplitude of finding a parton of a pion (moving with momentum p in a light cone frame) with parallel momentum equal to $x\mathbf{p}$ and transverse momentum less than Q. This wave function incorporates non-perturbative physics and plays an important role in exclusive hard scattering processes and in non-leptonic decays of heavy mesons. One example of application is the electromagnetic form factor of the pion, $F(q^2)$, defined by

$$\langle\pi(\mathbf{p}')|\bar{q}\gamma_\mu q|\pi(\mathbf{p})\rangle = F((p-p')^2)(p+p')_\mu \qquad (1)$$

This form factor can, in fact, be written in terms of ϕ and T_H (the perturbative scattering amplitude for the constituents) as

$$F(Q^2)=\int \phi^\dagger(x,Q^2)T_H(x,y,Q^2)\phi(y,Q^2)\mathrm{d}x\mathrm{d}y \qquad (2)$$

More formally the light cone wave function can be defined as [1] [2]

$$\phi^{ab}_{\alpha\beta}(x,Q^2) = F.T.\langle 0|T\{q^a_\alpha(z_1),\bar{q}^b_\beta(z_2)\}|\pi\rangle \qquad (3)$$

where F.T. indicates a Fourier transform on z_1 and z_2 assuming

- the sum of the parallel components of the momenta of the two partons is equal to the total pion momentum p.
- the transverse components have been integrated out up to momentum Q.

The n-th moment of ϕ is defined as

$$\langle\xi^n\rangle = \int_0^1 \xi^n\phi(\xi,Q^2)\mathrm{d}\xi \qquad (4)$$

In a typical lattice determination of such quantity the cut-off is provided by the lattice spacing, $Q \simeq a^{-1}$.

We finally wish to remark that $\langle\xi^0\rangle = 1$ is fixed by a normalization condition and $\langle\xi^1\rangle = 0$ because the wavefunction is symmetric under G-parity. Therefore $\langle\xi^2\rangle$ is the first non trivial moment of ϕ.

This second moment can be related to

$$\langle 0|O_{\mu\nu\rho}|\pi(\mathbf{p})\rangle = f_\pi\langle\xi^2\rangle p_\mu p_\nu p_\rho + \dots \qquad (5)$$

where the ellipsis indicates divergent terms,

$$O_{\mu\nu\rho} = \bar{q}\gamma_\mu\gamma_5 \overleftrightarrow{D}_\nu\overleftrightarrow{D}_\rho\, q' \qquad (6)$$

and

$$q\,\overleftrightarrow{D}\,q' \overset{def}{=} q\,\overrightarrow{D}\,q' + q\,\overleftarrow{D}\,q' \qquad (7)$$

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2. COMPUTATION

Our computation was carried on 154 quenched gauge configurations generated by the UKQCD collaboration using the Wilson gauge action. We use the Clover $O(a^2)$ improved action for the light quarks.

The parameters of our computation are:

- Volume equal to 24^3 x 48.
- β = 6.2 which corresponds to an inverse lattice spacing of $a^{-1} = 2.67 \pm 0.10$GeV.
- κ values 0.13460, 0.13510, 0.13530 (corresponding to pseudoscalar masses of 748,574 and 490MeV respectively)
- κ_{crit} = 0.13582 and c_{SW}=1.61

For technical reasons we choose not to improve the operators and do not smear the light quarks. The latter choice is motivated by the fact that the local axial current seems to have better superposition with the pion than other smeared operators we tried.

In order to calculate any moment of the pion light-cone wavefunction, we are required to study the matrix elements of lowest twist local operators between pion and vacuum. In a continuum world these operators are classified by their representation under the group of Lorentz, parity and charge conjugation. In the lattice-discretized world the Lorentz group is broken to $\mathcal{H}_4 \in$ O(4), the hypercubic group. Hence lowest twist local operators can mix with higher dimensional operators and introduce power divergences.

We choose a particular combination of the lattice operators that:

- transforms under an irreducible representation of the hypercubic group
- does not mix with higher dimensional operators.

In particular we choose $O_{\mu[\nu,\rho]}$ with $\mu \neq \nu \neq \rho \neq \mu$. which transforms as $(\frac{1}{2},\frac{1}{2}) \otimes \mathbf{8}^+$ under $\mathcal{H}_4$. The $\mathbf{8}^+$ irreducible representation would naively generate a term proportional to

$$p_\mu \left[\frac{p_\nu^2 + p_\rho^2}{2} - (\epsilon^{\mu\nu\rho\sigma} p_\sigma)^2\right] \quad (8)$$

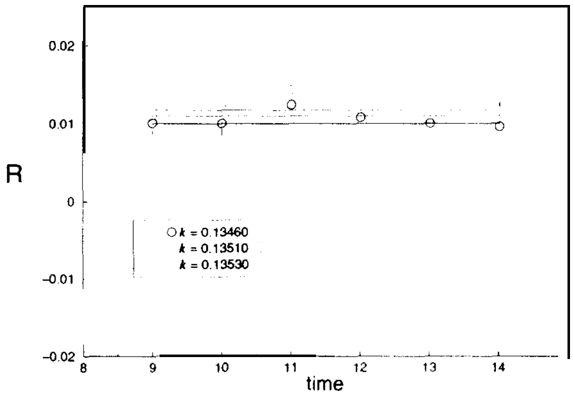

Figure 1. The Plot shows R as function of t, the timeslice, and κ. The best fits are included (for each value of κ).

This contribution vanishes for our matrix elements because of its parity. We introduce the following definition:

$$R = \frac{C_2^O(t,\mathbf{p})}{p_1 p_2 C_2^A(t,\mathbf{p})}_{\mathbf{p}=(1,1,0)} \quad (9)$$

where $C_2^Q(t,\mathbf{p})$ is the spatial Fourier transform (at momentum p) of $\langle Q(x)\bar{q}\gamma_5\psi q'(0)\rangle$, 0 is a short hand notation for $O_{\mu[\nu,\rho]}$ and $A = \bar{q}\gamma_5 q'$ is the usual axial current. For large $t = x_0 \to \infty$, C_2 (t) asymptotically approaches

$$C_2 \simeq \frac{Z_A}{2E(\mathbf{p})}\langle 0|Q(0)|\pi(\mathbf{p})\rangle e^{-E(\mathbf{p})t} \quad (10)$$

Using eq. (5), eq. (9) reduces to

$$R \simeq \langle\xi^2\rangle^{lattice} \frac{Z^A}{Z^O}\langle\xi^2\rangle^{\overline{\mathrm{MS}}}. \quad (11)$$

the desired second moment (without divergences) times a corrective matching factor to be determined perturbatively.

3. MATCHING

A big part of this work was the determination of the matching factor Z^O for 0 = $O_{\mu\nu\rho}$. The computation was performed assuming a gluon mass as regulator and on-shell external quarks with non-zero momentum. All 41 relevant diagrams were expanded in Taylor up to second order in p, compatibly with eq. (9).

As result of our computation we find that

$$\frac{Z^O}{Z^A} = 1.518 \tag{12}$$

and the coefficients are evaluated at $\alpha_s(q^*)$ for $q^* = 2/a$. We tried varying q^* by factors of two and this gives us an estimate of the error in the matching of the order of 10% that we include in our final result.

4. RESULTS

The result of our calculation is

$$\langle\xi^2\rangle^{lattice}_{Q=2.67GeV} = 0.185 \pm 0.032 \tag{13}$$

$$\langle\xi^2\rangle^{\overline{MS}}_{Q=2.67GeV} = 0.280 \pm 0.049^{+0.030}_{-0.013} \tag{14}$$

(the first error is statistical and the second is systematic due to matching, quenching error is not included). These numbers should be compared with results from sum rules

$$\langle\xi^2\rangle^{\overline{MS}}_{Q=5GeV} = 0.40 \pm 0.05 \tag{15}$$

with preceding independent lattice results

$$\langle\xi^2\rangle^{lattice}_{Q\simeq 1GeV} = 1.37 \pm 0.20\ [3] \tag{16}$$

$$\langle\xi^2\rangle^{lattice}_{Q\simeq 1GeV} = 0.25 \pm 0.10\ [4] \tag{17}$$

$$\langle\xi^2\rangle^{lattice}_{Q=2.4GeV} = 0.10 \pm 0.12\ [5] \tag{18}$$

and the exact asymptotic value

$$\langle\xi^2\rangle_{Q=\infty} = 0.2 \tag{19}$$

(confirmed by the Fermilab experiment E791, performed at $Q \simeq 3 - 4GeV$).

We find that our value for the second moment is smaller than sum rule predictions and is closer to the asymptotic value.

On the one side we conclude that the present computation has better control of statistical and perturbative errors than previous computations. On the other side we strongly feel that dynamical quarks may be playing an important role in this process and quenching errors are yet to be estimated and removed. We also believe that the same analysis should be repeated for different values of the lattice spacing in order to study the Q^2 dependence of $\langle\xi^2\rangle$.

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