

Hadronic Decays of Excited Heavy Mesons

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We studied the hadronic decays of excited states of heavy mesons (D , D_s , B and B_s) to lighter states by emission of π , η or K . Wavefunctions and energy levels of these excited states are determined using a Dirac equation for the light quark in the potential generated by the heavy quark (including first order corrections in the heavy quark expansion). Transition amplitudes are computed in the context of the Heavy Chiral Quark Model.

1. INTRODUCTION

In the context of the most general Quark Model, a heavy-light meson (H) is modeled with a light quark (q) bound to a static source of chromo-electro-magnetic field (the heavy quark h). Its hadronic transitions can be computed assuming that only the light quark enters in the reaction through and effective coupling g_A (Heavy Chiral Quark Model [1]).

Our work generally follows that of ref. [2–5]. We differ in the choice of the potential. Moreover, we included for the first time the mixing effects in the spectrum and decay amplitudes.

2. SPECTRUM

2.1. Notation

The Dirac wavefunction, Ψ , of the light quark can be determined by solving the eigenvalue problem

$$H\Psi_{n,\ell,j,J,M} = E_{n,\ell,j,J}\Psi_{n,\ell,j,J,M} \quad (1)$$

where H is the Hamiltonian of the system and Ψ is 4-spinor that represents the wavefunction of the system. In our notation

$$\Psi_{n\ell j J M} = C_{j,m,\frac{1}{2},S}^{J,M} \xi_S \begin{pmatrix} f_{n\ell j}^0 k_{\ell,j,m}^+ Y_{m-\frac{1}{2}}^\ell \\ i f_{n\ell j}^0 k_{\ell,j,m}^- Y_{m+\frac{1}{2}}^\ell \\ f_{n\ell j}^1 k_{2j-\ell,j,m}^+ Y_{m-\frac{1}{2}}^{2j-\ell} \\ f_{n\ell j}^1 k_{2j-\ell,j,m}^- Y_{m+\frac{1}{2}}^{2j-\ell} \end{pmatrix} \quad (2)$$

where n is the radial quantum number, ℓ is the orbital quantum number, j, m are the total spin

of the light quark and its z component, J, M are the total spin of the meson and its z component, f^0, f^1 are radial wavefunctions and Y_m^ℓ are the usual spherical harmonics. ξ_S is the 2-spinor associated to the heavy quark h and S is its spin.

Our convention for the phase and the normalization is such that

$$k_{\ell,j,m}^\pm = \begin{cases} \sqrt{\frac{\ell \pm m + \frac{1}{2}}{2\ell+1}} & \text{if } j = \ell + \frac{1}{2} \\ \pm \sqrt{\frac{\ell \mp m + \frac{1}{2}}{2\ell+1}} & \text{if } j = \ell - \frac{1}{2} \end{cases} \quad (3)$$

2.2. Choice of the potential

Ignoring $1/m_h$ corrections, the most general form of the Hamiltonian that appear in eq. (1) is

$$H^{(0)} = -i\gamma^0 \gamma^i \partial_i + \gamma^0 m_q + \gamma^0 V_s + M_h + V_v \quad (4)$$

where V_s is a spin independent potential, V_v is a spin dependent potential and M_h is a total energy shift (not to be confused with the mass of the heavy quark m_h that appear in corrections to $H = H^{(0)} + O(1/m_h)$).

Asymptotic freedom suggests that at short distances $V_v \simeq 1/r$ dominates, while lattice simulations suggest that at large distances $V_s \simeq r$ dominates. As it was observed in ref. [2], the choice of Coulomb-like potential at short distance is inconsistent with $1/m_h$ spin-dependent correction to H^0 , because of ultraviolet divergences. The solution of the problem is that, in the context of the Dirac equation with a finite m_h , it is not correct to localize the heavy quark with a delta function since one must take into account the spatial degrees of freedom of the heavy quark. Our pragmatic approach to the problem is that of delocalizing the heavy quark within a length scale $1/\lambda$

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assuming a Gaussian wave-function, $\Phi(x)$, for the former. The effective potential felt by the light quark is, therefore, a convolution of the Coulomb-like potential with the square of the wave-function of the quark:

$$V_v(r) = \int |\Phi(\mathbf{x})|^2 \frac{\alpha_s}{|\mathbf{r} - \mathbf{x}|} d^3\mathbf{x} = \frac{\alpha_s}{r} \text{erf}(\lambda r) \quad (5)$$

Our choice for the spin-independent part of the potential is

$$V_s(r) = br + c \quad (6)$$

(notice from eq. (4) that c is not a physical parameter since it can be re-absorbed into the definition of m_q).

2.3. $1/m_h$ correction

We solve the eigenvalue problem associated to the eq. (1) using the Hamiltonian in eq. (4) and the potentials (5,6). In this way we determine the radial wave-functions $f_{n\ell}^0$, $f_{n\ell}^1$ and the associated eigenvalues $E_{n\ell j}$.

$1/m_h$ corrections to the the Hamiltonian have been derived in [4] in the Bethe-Salpeter formalism. They are responsible for the spin-orbit interaction, the hyperfine splitting and the mixing of states with the same j . We include these effects as a perturbation to the energy levels ($\delta E_{n\ell j}$) and also determine the mixing coefficients for each doublet of states.

In terms of the f^i the $1/m_h$ correction to the energy levels reads as

$$\delta E = \int (A + B + C) r^2 dr \quad (7)$$

where one can identify the contributions proportional to p^2 :

$$A = - \sum_i f^i \left(\partial_r^2 + \frac{2}{r} \partial_r - \frac{\ell^2 + \ell}{r^2} \right) f^i \quad (8)$$

the spin orbit interaction, for $j = \ell + \frac{1}{2}$:

$$B = V_v \left[f^1 \left(\partial_r - \frac{\ell}{r} \right) f^0 - f^0 \left(\partial_r + \frac{\ell+2}{r} \right) f^1 \right] \quad (9)$$

for $j = \ell - \frac{1}{2}$:

$$B = V_v \left[f^1 \left(\partial_r + \frac{\ell+1}{r} \right) f^0 - f^0 \left(\partial_r - \frac{\ell-1}{r} \right) f^1 \right] \quad (10)$$

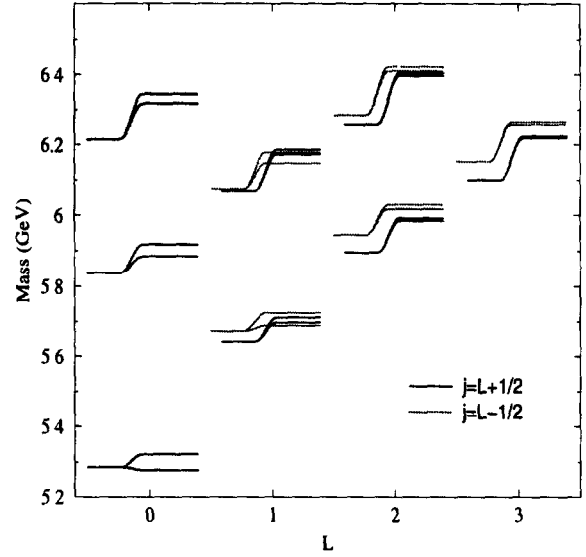


Figure 1. Spectrum of excited B mesons.

Particle	$n^j \ell_J$	Exp.	Model	$\Gamma_{\text{tot}}/g_A^2$
D	$1^{\frac{1}{2}} S_0$	1864	1871	—
D^*	$1^{\frac{1}{2}} S_1$	2007	2006	—
D_1	$1^{\frac{3}{2}} P_1$	2427	2420	25.0
D_2^*	$1^{\frac{3}{2}} P_2$	2459	2462	38.2
D_s	$1^{\frac{1}{2}} S_0$	1969	1965	—
D_s^*	$1^{\frac{1}{2}} S_1$	2114	2112	—
D_{s1}	$1^{\frac{3}{2}} P_1$	2535	2535	94.3
D_{s2}^*	$1^{\frac{3}{2}} P_2$	2573	2579	4.9
B	$1^{\frac{1}{2}} S_0$	5279	5278	—
B^*	$1^{\frac{1}{2}} S_1$	5325	5322	—
B_0	$1^{\frac{1}{2}} P_0$		5689	174.6
B_1	$1^{\frac{3}{2}} P_1$		5698	14.3
B_1^*	$1^{\frac{1}{2}} P_1$		5725	168.0
B_2^*	$1^{\frac{3}{2}} P_2$		5712	20.0
B'	$2^{\frac{1}{2}} S_0$		5885	28.1
B'^*	$2^{\frac{1}{2}} S_1$		5918	49.3
B_s	$1^{\frac{1}{2}} S_0$	5369	5369	—
B_s^*	$1^{\frac{1}{2}} S_1$	5416	5417	—
B_{s1}	$1^{\frac{3}{2}} P_1$		5801	—
B_{s2}^*	$1^{\frac{3}{2}} P_2$		5815	0.2

Table 1. Tabulated spectrum for the observed states used in the fit together with predictions for some excited B states. All units are in MeV.

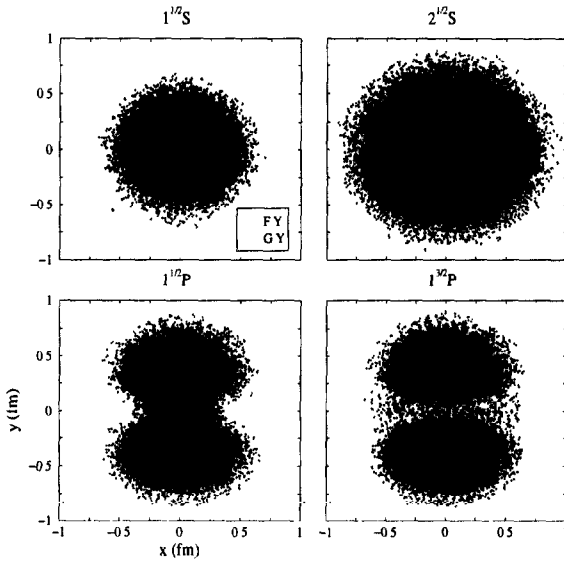


Figure 2. Orbitals for some excited B mesons.

and the hyperfine splitting contribution:

$$C = (-1)^{J-J'} \frac{2j+1}{2J+1} V'_v f^0 f^1 \quad (11)$$

2.4. Choice of parameters and predictions

The nine parameters of the model (α_s , λ , b , $m_q|_{q=u}$, $m_q|_{q=s}$, $m_h|_{h=c}$, $M_h|_{h=c}$, $m_h|_{h=b}$, $M_h|_{h=b}$) are determined by best fit (minimum χ^2) to the known spectrum of excited states of the D , D_s , B and B_s mesons. Our results are:

$$\begin{aligned} \alpha_s &= 0.339 \\ \lambda &= 2.820 \text{ GeV} \\ b &= 0.257 \text{ GeV} \\ m_u &= 0.073 \text{ GeV} & m_s &= 0.214 \text{ GeV} \\ m_c &= 1.52 \text{ GeV} & M_c &= 1.52 \text{ GeV} \\ m_b &= 4.67 \text{ GeV} & M_b &= 4.68 \text{ GeV} \end{aligned}$$

The spectrum for some of the states plotted in fig. 1 and tabulated in table 1. A density plot of $|f_{n\ell}^0 Y_0^\ell|^2$ and $|f_{n\ell}^1 Y_0^{2j-\ell}|^2$ for some excited states of a B meson is reported in fig. 2.

2.5. Tests of the model

Despite the good fit of the mass spectrum we decided to test our model by comparing some of

the transition amplitudes with some recent lattice results. In particular we computed

$$\mathcal{A}_{B^* \rightarrow B\pi}(r) = \int \langle B^* | A_\mu(\mathbf{r}) | B \rangle d\Omega_r \quad (12)$$

using our chiral quark model (where the only unknown parameter is the overall normalization, g_A , which is the effective coupling of the quark to the axial current, A_μ) and comparing it with the lattice result of ref. [6]. The comparison is shown in fig. 3. In the plot the point at $r = 0$ is used to fix the relative normalization. A more sophisticated analysis of the lattice results gives² $g_A = 0.42 \pm 0.09$.

3. HADRONIC DECAYS

3.1. Decay amplitudes

We consider here the most general hadronic transition

$$H' \rightarrow H + x \quad (13)$$

where H' is a heavy meson with associated wave-function Ψ' , H is a heavy meson with associated wave-function Ψ and x is a light meson with momentum \mathbf{p} . Such a transition is mediated by a matrix element of the form

$$I(X, \mathbf{p}) = g_A \int \bar{\Psi}^* X e^{-i\mathbf{p}\cdot\mathbf{r}} \Psi' d^3r \quad (14)$$

where X is the 4×4 spin matrix that characterizes the transition. In the particular case in which the light meson x is a pseudoscalar (π , η or K) $X = \not{p}$ ⁵, while if the light meson x is a vector (ρ , ω or K^*) $X = \not{\epsilon}$ (and ϵ is the polarization vector of the outgoing vector meson).

The exponential can be expanded in products of spherical harmonics and spherical Bessel functions (j_k), thus giving

$$I(X, \mathbf{p}) = \sum_{\ell_x, m_x} Y_{\ell_x}^{\ell_x*}(\hat{\mathbf{p}}) C_{J, M; \ell_x, m_x}^{J', M'} \mathcal{A}_{\ell_x}(X, p) \quad (15)$$

We computed $\mathcal{A}_{\ell_x}(X, p)$ for a complete set of spin matrices X and proved that it can always be reduced to integrals of the form

$$\mathcal{A}_{\ell_x}(X, p) = g_A \sum_{i, j=0,1} c_{\ell_x}^{ijk}(X) \int_0^\infty (f^i j_k f^j) r^2 dr \quad (16)$$

²this is a preliminary determination valid in the limit $p_\pi, m_\pi \rightarrow 0$

where $c_{\ell_x}^{ijk}(X)$ depend also on the quantum numbers of the mother and the daughter mesons but not on the radial wavefunctions.

3.2. Decay widths

The decay width for transition of eq. (13), when light meson x is emitted in an eigenstate of the total momentum p and of the angular momentum ℓ_x , is given by

$$\Gamma_{\ell_x}^{H'Hx} = \frac{\zeta^2}{16\pi^2 f_\pi^2} \frac{2J+1}{2J'+1} \frac{M}{M'} |A_{\ell_x}(p\gamma^5, p)|^2 \quad (17)$$

where $\zeta = \sqrt{3}$, $1/\sqrt{3}$, $2/\sqrt{3}$ or 1 for π , η (for a nonstrange heavy meson), η (for a strange heavy meson) or K respectively; M' , J' and the mass and total angular momentum of the mother meson H' ; M , J are the mass and angular momentum of the daughter meson H , and p can be determined by energy-momentum conservation ($p_0 = M' - M$, $p = |\mathbf{p}| = \sqrt{p_0^2 - m_x^2}$). The total decay width and the branching ratios are defined as

$$\Gamma_{\text{tot}}^{H'} = \sum_{x, \ell_x} \Gamma_{\ell_x}^{H'Hx}; \quad Br(H' \rightarrow Hx; \ell_x) = \frac{\Gamma_{\ell_x}^{H'Hx}}{\Gamma_{\text{tot}}^{H'}} \quad (18)$$

where the sum on x spans all the hadronic decay modes with emission of a light pseudoscalar meson. The total width for some of the mesons is reported in table 1.

4. CONCLUSIONS

We computed the spectrum and the width of hadronic decays of excited D , D_s , B and B_s mesons in the context of the chiral quark model. As an example, we report here some hadronic decay channels for the first radial excited B meson (not including the ρ decays)

$$\begin{aligned} B(2\frac{1}{2}S_0) & \\ \hookrightarrow B(1\frac{1}{2}S_1) + \pi & \quad (\ell_\pi = 1 \quad Br = 77\%) \\ \hookrightarrow B(1\frac{1}{2}P_0) + \pi & \quad (\ell_\pi = 0 \quad Br = 22\%) \\ \hookrightarrow B(1\frac{1}{2}S_0) + \pi & \quad (\ell_\pi = 0 \quad Br = 100\%) \\ \hookrightarrow B(1\frac{3}{2}P_2) + \pi & \quad (\ell_\pi = 2 \quad Br = 0.33\%) \\ \hookrightarrow B(1\frac{1}{2}S_1) + \eta & \quad (\ell_\eta = 1 \quad Br = 0.03\%) \end{aligned}$$

More complete tables will be published on a separate paper.

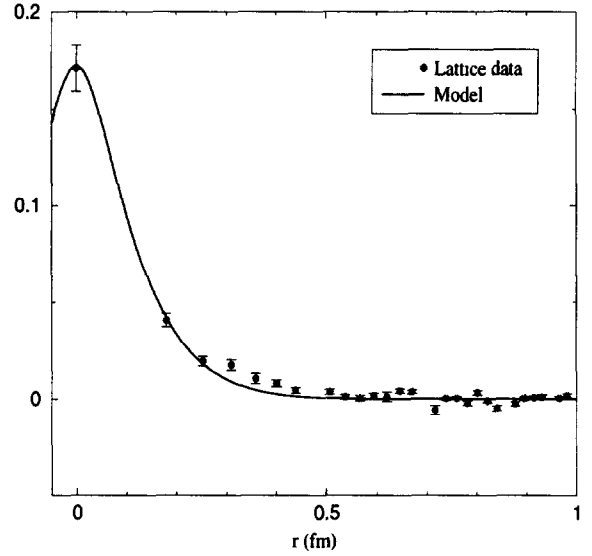


Figure 3. Comparison between a prediction of our model and the lattice QCD result [6].

We finally remark how our model is able to fit the masses of observed excited states within less than 10 MeV discrepancy (within 4 MeV in average) better than was done in preceding works.

This work was performed at Fermilab, a U.S. Department of Energy Lab (operated by the University Research Association, Inc.), under contract DE-AC02-76CHO3000.

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