Excited heavy-light systems and hadronic transitions

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A detailed study of orbital and radial excited states in D,D_s,B, and B_s systems is performed. The chiral quark model provides the framework for the calculation of pseudoscalar meson ($\pi,K,\ldots$) hadronic transitions among heavy-light excited and ground states. To calculate the excited states masses and wave functions, we must resort to a relativistic quark model. Our model includes the leading order corrections in $1/m_{(c,b)}$ (e.g., mixing). Numerical results for masses and light hadronic transition rates are compared to existing experimental data. The effective coupling of the chiral quark model can be determined by comparing with independent results from lattice simulations ($g_A^8=0.53\pm0.11$) or fitting to known widths ($g_A^8=0.82\pm0.09$).

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I. INTRODUCTION

Although heavy quark spectroscopy is now a rather mature subject, a number of interesting issues remain. In particular, the detailed properties of the excitation spectrum of heavy-light mesons (D,D_s,B,B_s) and their light hadronic transitions are yet to be fully understood. Experimentally, much of this excitation spectrum remains to be observed. Only the ground state S waves and a few of the $j_l=3/2$ P waves are presently well established. However, many of these states will be accessible in the present and future B factories: CLEO, BaBar, Belle, Collider Detector at Fermilab (CDF), D0, BTeV, and CERN Large Hadron Collider (LHC B). In addition to furthering our understanding of QCD dynamics, the detailed study of these excited states may have practical benefits. For example, triggering on excited states may provide an efficient method of same side B tagging in hadron colliders. Tagging is essential to the study of CP violation in the B system.

The theoretical tools available to determine the properties of excited states in heavy-light systems include heavy quark effective theory (HQET) and low energy chiral effective theory [1]. Unfortunately, these tools are not sufficient to determine the detailed properties of these states. Lattice gauge theory is the only existing technique that allows the systematic study of all the aspects of QCD in heavy-light systems. Detailed studies of the P-wave excited heavy-light states within the quenched approximation already exist [2]. Future lattice studies will provide more insight into the nature of QCD dynamics as well as the masses and static properties of heavy-light hadrons.

It is clear that a model to estimate the hadronic transitions from excited state to ground states would also be very useful. Such a formalism has been developed and applied extensively to transitions in heavy-heavy ($\bar{Q}Q'$) systems [3]. However, heavy-light mesons are more difficult because the light quark is subject to the full nonperturbative QCD dynamics. One possible approach to providing a framework for these hadronic transitions is to use the chiral quark model [4]. This has been suggested by Goity and Roberts [5].

In this paper we closely follow the work done in Refs. [5–7]. To compute the masses and wave functions of the excited states we use a Dirac equation for the light quark in the potential generated by the heavy quark (including first order corrections in the heavy quark expansion and mixing effects). We then use these masses and wave functions to compute the hadronic decay amplitudes of excited heavy mesons in the context of a chiral quark model [4].

The main differences between the present and preceding works are in the choice for the parameters of the chromoelectric potential and in the inclusion of mixing effects both in the spectrum and in the decay amplitudes. Moreover, we use our results for the radial wave functions of the excited mesons to make a comparison with recent lattice results. From the comparison we extract an estimation for g_A^8, the effective coupling of the quark to the pseudoscalar mesons. We find $g_A^8=0.53\pm0.11$.

We present numerical results for the low-lying spectrum (excited states up to the $3S$ states). We also compute the pseudoscalar meson hadronic transitions for these states as a function of the chiral quark model effective coupling constant. Comparing our results with recent experimental width measurements we estimate this effective coupling $g_A^8=0.82\pm0.09$.

In Sec. II we discuss our determination of the spectrum of excited states. Our notation, the choice of the potential, inclusion of mixing and other order $1/m_h$ corrections are explained. Details of the masses and wave functions are presented for the low-lying excitation spectrum. A comparison is made with the present experimental data. Our treatment of hadronic decays is described in Sec. III. The analytic results are summarized in Eqs. (31)–(33). Explicit expressions for the coupling coefficients appearing in these equations are given in Appendix A. Also in Sec. III, details of the partial rates for the $1S$ and $1P$ states in the D,D_s,B and B_s systems are presented. Again comparison is made with the present experimental data. A complete list of the remaining results for masses and partial decay widths is reported in Appendix B.

II. SPECTRUM

A. Basic model and notation

The general Hamiltonian of the heavy-light system can be expanded in powers of $(1/m_h)$

$$\mathcal{H}=\mathcal{H}^{(0)}+\frac{1}{m_h}\mathcal{H}^{(1)}+\frac{1}{m_h^2}\mathcal{H}^{(2)}+\ldots. \tag{1}$$

However, even within the heavy quark limit, the general form of the zeroth order Hamiltonian, $\mathcal{H}^0$, still involves the full nonperturbative QCD dynamics for the remaining degrees of freedom (including light quark pair creation and gluonic degrees of freedom). At present it cannot be solved analytically. We are forced to resort to use a relativistic potential model for $\mathcal{H}^0$.

We model the most general heavy-light meson (in the D,D_s,B,B_s family), H, as a bound state of a light quark (q) and a heavy quark (h). The heavy quark is treated as a static source of chromoelectric field and the only quantum number associated with it is its spin. The light quark is treated relativistically and its state is described by the wave function $\psi_{n,l,j,m}(r,\theta,\varphi)$. In analogy with the hydrogen atom, we introduce the following quantum numbers:

n, the number associated with the radial excitations;
l, the orbital angular momentum;
j, the total angular momentum of the light quark;
m, the component of j along the $\hat{z}$ axis;
J, the total angular momentum of the system;
M, the component of J along the $\hat{z}$ axis;
S, the spin of the heavy quark along the $\hat{z}$ axis.

The parameters of our model are the masses of the light quarks (m_q for $q=u,d$ or s), the masses of the heavy quarks (m_h for $h=c$ or b) and the chromoelectric potential of the heavy quark $[V(r)]$.

The total wave function of the system can be decomposed as follows:

$$\Psi_{n,l,j,J,M}(r,\theta,\varphi) = \sum_{S\in\{-1/2,+1/2\}} C^{J,M}_{j,m;1/2,S}\psi_{n,l,j,m}(r,\theta,\varphi)\otimes\xi_S \tag{2}$$

where $C^{J,M}_{j,m;1/2,S}$ are the usual Clebsch-Gordan coefficients and ξ_S is a two component spinor representing the heavy quark. Equation (2) is a solution of the following eigenvalue problem:

$$\mathcal{H}\Psi_{n,l,j,J,M}=E_{n,l,j,J}\Psi_{n,l,j,J,M} \tag{3}$$

where $\mathcal{H}$ is the Hamiltonian of the system. The energy levels in Eq. (3) do not depend on M because of rotational invariance.

We rewrite Eq. (2) introducing the most general parametrization for the four spin components of the light quark wave function

$$\Psi_{n,l,j,J,M}(r,\theta,\varphi)=\sum_{S\in\{-1/2,+1/2\}} C^{J,M}_{j,m;1/2,S} \times\begin{pmatrix} if^0_{n,l,j}(r)k^+_{l,j,m}\ Y^l_{m-1/2}(\theta,\varphi)\\ if^0_{n,l,j}(r)k^-_{l,j,m}\ Y^l_{m+1/2}(\theta,\varphi)\\ f^1_{n,l,j}(r)k^+_{2j-l,j,m}Y^{2j-l}_{m-1/2}(\theta,\varphi)\\ f^1_{n,l,j}(r)k^-_{2j-l,j,m}Y^{2j-l}_{m+1/2}(\theta,\varphi)\end{pmatrix}\otimes\xi_S. \tag{4}$$

Here $Y^l_m(\theta,\varphi)$ are spherical harmonics that encode the angular dependence while $f^0_{n,l,j}(r)$, $f^1_{n,l,j}(r)$ are real functions that encode that radial dependence. $k^+_{l,j,m}$ and $k^-_{l,j,m}$ are fixed, up to an overall phase, by imposing a normalization condition. Our choice of the phase is such that

$$k^{\pm}_{l,j,m}=\begin{cases} +\sqrt{\dfrac{l\pm m+\frac{1}{2}}{2l+1}} & \text{for } j=l+\dfrac{1}{2},\\ \pm\sqrt{\dfrac{l\mp m+\frac{1}{2}}{2l+1}} & \text{for } j=l-\dfrac{1}{2}.\end{cases} \tag{5}$$

B. Choice of the potential

Within our basic framework, $\mathcal{H}^{(0)}$ is given by the relativistic Dirac Hamiltonian

$$\mathcal{H}^{(0)}=\gamma^0(-i\partial\!\!\!/+m_q)+V(r) \tag{6}$$

and the rotational-invariant potential is the sum of a constant factor (M_h), a scalar part (V_s) and (the zeroth component of) a vector part (V_v)

$$V(r)=M_h+\gamma^0V_s(r)+V_v(r). \tag{7}$$

The constant M_h is a an overall energy shift that depends on the heavy quark flavor and, in general, it is not equal to m_h, as often assumed in the literature. For this reason we consider m_h and M_h two independent parameters of the model.

Asymptotic freedom suggests that at short distances the potential is dominated by a vector part that asymptotically approaches a Coulomb potential, $V\sim V_v\sim 1/r$. On the other hand, lattice simulations indicate that at large distances the potential is confining, scalar and asymptotically linear, $V\sim V_s\sim r$.

The naive assumption about a short distance Coulomb-like divergent behavior of the potential is doomed because it gives rise to ultraviolet divergences (as discussed in Ref. [8] and Ref. [9]). In this context the divergence arises in the $1/m_h$ correction to the energy and it is due to the inconsistency of a static point-like source (the heavy quark) within a relativistic framework. One solution is assuming that the heavy quark is static but not point-like, therefore the potential that it generates is a convolution of the Coulomb-like

potential and the square of the heavy quark wave function (peaked around the center of mass of the system and smeared within some small length scale λ^{-1}).

More generally, one is allowed to cure this divergence by regulating the potential close to the origin (on a length scale of the order λ^{-1}). Different choices for the regulator are allowed and they do not affect the physics we want to describe, providing that λ^{-1} is small enough. The values of the parameters that appear in the Hamiltonian, on the contrary, depend on this choice since they run with λ. In fact, to obtain the same spectrum, different choices for the regulator imply different fitting parameters.

We chose to regulate the vector potential by assuming a Gaussian shape for the wave function of the heavy quark, $\Phi(x)=\exp(-x^2\lambda^2/2)$, and with this choice

$$V_v(r)=-\frac{4}{3}\int |\Phi(x)|^2\frac{\alpha_s}{|\mathbf{r}-\mathbf{x}|}\mathrm{d}^3x=-\frac{4}{3}\frac{\alpha_s}{r}\mathrm{erf}(\lambda r). \quad (8)$$

For the scalar potential we assume a simple linear form

$$V_s(r)=br+c. \quad (9)$$

We observe that c is not a physical parameter since it can be absorbed into the definition of m_q. For this reason c will be omitted from now on.

Summarizing, the nine parameters of our model are

$$\alpha_s,\lambda,b,m_u,m_s,m_c,M_c,m_b,M_b \quad (10)$$

where $m_u\equiv m_d$ and m_s are mass parameters for the light u,d and s quarks, respectively, equivalent to constituent quark masses shifted by the constant amount c of Eq. (9), which is undetermined in our model. m_c is the mass of the c quark with M_c the corresponding energy shift and analogously for the b quark.

C. $1/m_h$ correction

For any given set of input parameters we solve the eigenvalue problem, Eq. (3), using the Hamiltonian of Eq. (6) and the potential specified by Eqs. (7), (8) and (9). In this way we determine the radial wave functions $f^0_{n,l,j}$ and $f^1_{n,l,j}$ associated with the energy levels $E^{(0)}_{n,l,j}$. We then compute $1/m_h$ corrections to the energy levels in first order perturbation theory

$$E_{n,l,j,J}=E^{(0)}_{n,l,j}+\frac{1}{m_h}\delta E^{(1)}_{n,l,j,J} \quad (11)$$

where, ignoring for the moment the mixing of the states,

$$\delta E^{(1)}_{n,l,j,J}=\sum_M \int \Psi^\dagger_{n,l,j,J,M}(x)\mathcal{H}^{(1)}\Psi_{n,l,j,J,M}(x)d^3x. \quad (12)$$

The analytical expression for $\mathcal{H}^{(1)}$ has been derived in Ref. [6] using the Bethe-Salpeter formalism. In terms of the radial wave functions (after the analytical integration of the angular part), we rewrite $\delta E^{(1)}$ as a sum of three contributions

$$\delta E^{(1)}_{n,l,j,J}=A_{n,l,j,J}+B_{n,l,j,J}+C_{n,l,j,J}. \quad (13)$$

These terms are, respectively, (i) the kinetic energy

$$A_{n,l,j,J}=-\frac{1}{2}\int_0^\infty\left[f^0_{n,l,j}\left(\partial_r^2+\frac{2}{r}\partial_r-\frac{l^2+l}{r^2}\right)f^0_{n,l,j}\right. \quad (14)$$

$$\left.\times f^1_{n,l,j}\left(\partial_r^2+\frac{2}{r}\partial_r-\frac{\overline{l}^2+\overline{l}}{r^2}\right)f^1_{n,l,j}\right]r^2dr \quad (15)$$

with $\overline{l}=2j-l$. (ii) A shift due to spin-orbit interaction

$$B_{n,l,j,J}=\int_0^\infty V_v\left[f^1_{n,l,j}\left(\partial_r-\frac{l}{r}\right)f^0_{n,l,j}-f^0_{n,l,j}\right.$$
$$\left.\times\left(\partial_r+\frac{l+2}{r}\right)f^1_{n,l,j}\right]r^2dr \quad (16)$$

for $j=l+\frac{1}{2}$, or

$$B_{n,l,j,J}=\int_0^\infty V_v\left[f^1_{n,l,j}\left(\partial_r+\frac{l+1}{r}\right)f^0_{n,l,j}-f^0_{n,l,j}\right.$$
$$\left.\times\left(\partial_r-\frac{l-1}{r}\right)f^1_{n,l,j}\right]r^2dr \quad (17)$$

for $j=l-\frac{1}{2}$. (iii) The hyperfine splitting

$$C_{n,l,j,J}=(-1)^{J-l}\frac{2j+1}{2J+1}\int_0^\infty(\partial_r V_v)f^0_{n,l,j}f^1_{n,l,j}\ r^2dr. \quad (18)$$

D. Mixing

In Eqs. (11) and (12) we assumed that the Hamiltonian was diagonal. This is not the case because the $1/m_h$ interaction term in the Hamiltonian mixes states. In general correction terms can mix any states with the same total angular momentum, J, and parity, P. However, there are only two types of sizable mixings. Large mixing can occur for pairs of states $\Psi_{n,l,j,J,M}$ and $\Psi_{n',l',j',J,M}$ with (1) $n=n'$, $l=l'$ and $j+\frac{1}{2}=j'-\frac{1}{2}=l=J$ (i.e., mixing within a given n and l multiplet) or (2) $n+1=n'$, $l+2=l'$ and $j+\frac{1}{2}=j'-\frac{1}{2}=l=J$ (e.g., S-D mixing). The off-diagonal term in the Hamiltonian that mixes such pairs of states has the form

TABLE I. Tabulated spectrum for D mesons. E^0 denotes the lowest order energies. $E^{\text{phys.}}$ includes all the order $1/m_h$ corrections. (All units are in GeV.) The mixings between lowest order states are denoted by ϕ.

$H(n^j L_J)$	$m_{\text{expt.}}$	E^0	$E^{\text{phys.}}$	ϕ (%)
$D(1^{1/2}S_0)$	1.865	1.895	1.868	
$D(1^{1/2}S_1)$	2.007	1.895	2.005	
$D(1^{1/2}P_0)$		2.282	2.377	
$D(1^{3/2}P_1)$	2.422	2.253	2.417	−10.92
$D(1^{3/2}P_2)$	2.459	2.253	2.460	
$D(1^{1/2}P_1)$		2.282	2.490	10.92
$D(2^{1/2}S_0)$		2.447	2.589	
$D(2^{1/2}S_1)$		2.447	2.692	2.17
$D(1^{5/2}D_2)$		2.504	2.775	−5.41
$D(1^{3/2}D_1)$		2.553	2.795	−2.17
$D(1^{5/2}D_3)$		2.504	2.799	
$D(1^{3/2}D_2)$		2.553	2.833	5.41
$D(2^{1/2}P_0)$		2.683	2.949	
$D(2^{3/2}P_1)$		2.679	2.995	−10.70
$D(2^{3/2}P_2)$		2.679	3.035	1.79
$D(2^{1/2}P_1$		2.683	3.045	10.70
$D(1^{7/2}F_3)$		2.709	3.074	−3.17
$D(1^{7/2}F_4)$		2.709	3.091	
$D(1^{5/2}F_2)$		2.760	3.101	−1.79
$D(1^{5/2}F_3)$		2.760	3.123	3.17
$D(3^{1/2}S_0)$		2.823	3.141	
$D(3^{1/2}S_1)$		2.823	3.226	

$$\epsilon = \frac{1}{m_h}\sum_M \int \Psi^\dagger_{n,l,j,J,M}(x)\mathcal{H}^{(1)}\Psi_{n',l',j',J,M}(x)d^3x$$
$$= (-1)^{J-l}\frac{1}{m_h}\frac{\sqrt{J(J+1)}}{2J+1}\int_0^\infty (\partial_r V_v)[f^0_{n,l,j}f^1_{n',l',j'} + f^1_{n,l,j}f^0_{n',l',j'}]r^2 dr \tag{19}$$

and it induces a mixing in the wave functions and in the energy levels

$$\begin{pmatrix}\psi_{n,l,j,m}\\ \psi_{n',l',j',m}\end{pmatrix}^{\text{phys}} = \begin{pmatrix} 1 & +\frac{\epsilon}{2\Delta} \\ -\frac{\epsilon}{2\Delta} & 1\end{pmatrix}\begin{pmatrix}\psi_{n,l,j,m}\\ \psi_{n',l',j',m}\end{pmatrix} + \mathcal{O}(\epsilon^2) \tag{20}$$

$$\begin{pmatrix}E_{n,l,j,J}\\ E_{n',l',j',J}\end{pmatrix}^{\text{phys}} = \begin{pmatrix} E_{n,l,j,J} + \frac{\epsilon^2}{2\Delta} \\ E_{n',l',j',J} - \frac{\epsilon^2}{2\Delta}\end{pmatrix} + \mathcal{O}(\epsilon^2) \tag{21}$$

with $\Delta = (E_{n,l,j,J} - E_{n',l',j',J})/2$.

We find that the effect of mixing is generally negligible except for the P waves, where the mixing among wave functions can be of the order of 10%. In Tables I–IV we report the value of $\phi = (100/2)\epsilon/\Delta$ for each excited state. It measures, in percent, the contribution of the mixing to the wave function.

E. Determination of the parameters and predictions

The nine parameters of our model, Eq. (10), are determined numerically as follows: We define a function $\mathcal{F}$ of the input parameters that finds eigenvalues and eigenfunctions of Eq. (3) using a fourth order Runge-Kutta formula, corrects the energy levels by including the $1/m_h$ perturbative corrections (including mixing effects) and returns

$$\chi^2 = \sum_{\text{observed states}} \left(\frac{E^{\text{phys.}}_{n,l,j,J} - m_{n,l,j,J}}{\delta m_{n,l,j,J}}\right)^2 \tag{22}$$

where $E^{\text{phys.}}_{n,l,j,J}$ are the computed energy levels and $m_{n,l,j,J} \pm \delta m$ are the measured masses (with their experimental error) of the corresponding particles.

We then minimize $\mathcal{F}$ in its nine dimensional domain. We repeat this procedure with different sets of starting parameters until we are confident that we have found the absolute minimum. The experimental data used for the "observed states" in the fit are reported in the third column of Tables I–IV.

Our best fit gives the following values for the parameters:

$$\begin{aligned} \alpha_s &\quad 0.339 \\ \lambda &\quad 2.823\ \text{GeV} \\ b &\quad 0.257\ \text{GeV}^2 \\ m_u &\quad 0.071\ \text{GeV} \\ m_s &\quad 0.216\ \text{GeV} \\ m_c &\quad 1.511\ \text{GeV} \\ M_c &\quad 1.292\ \text{GeV} \\ m_b &\quad 4.655\ \text{GeV} \\ M_b &\quad 4.685\ \text{GeV}. \end{aligned} \tag{23}$$

The corresponding predicted spectrum is reported in Fig. 1 and in the fifth column of Tables I–IV. The best fit parameters reported here differ slightly from those reported in Ref. [10] because we use here most recent value for $m_{n,l,j,J}$.

TABLE II. Tabulated spectrum for D_s mesons. (All units are in GeV.) The notation is as in Table I.

$H(n^jL_J)$	$m_{\text{expt.}}$	E^0	$E^{\text{phys.}}$	ϕ (%)
$D_s(1^{1/2}S_0)$	1.969	1.988	1.965	
$D_s(1^{1/2}S_1)$	2.112	1.988	2.113	
$D_s(1^{1/2}P_0)$		2.374	2.487	
$D_s(1^{3/2}P_1)$	2.535	2.353	2.535	−11.62
$D_s(1^{3/2}P_2)$	2.573	2.353	2.581	
$D_s(1^{1/2}P_1)$		2.374	2.605	11.62
$D_s(2^{1/2}S_0)$		2.540	2.700	
$D_s(2^{1/2}S_1)$		2.540	2.806	1.97
$D_s(1^{5/2}D_2)$		2.606	2.900	−6.11
$D_s(1^{3/2}D_1)$		2.648	2.913	−1.97
$D_s(1^{5/2}D_3)$		2.606	2.925	
$D_s(1^{3/2}D_2)$		2.648	2.953	6.11
$D_s(2^{1/2}P_0)$		2.777	3.067	
$D_s(2^{3/2}P_1)$		2.775	3.114	−10.58
$D_s(2^{3/2}P_2)$		2.775	3.157	1.81
$D_s(2^{1/2}P_1)$		2.777	3.165	10.58
$D_s(1^{7/2}F_3)$		2.812	3.203	−3.60
$D_s(1^{7/2}F_4)$		2.812	3.220	
$D_s(1^{5/2}F_2)$		2.857	3.224	−1.81
$D_s(1^{5/2}F_3)$		2.857	3.247	3.60
$D_s(3^{1/2}S_0)$		2.917	3.259	
$D_s(3^{1/2}S_1)$		2.917	3.345	

TABLE III. Tabulated spectrum for B mesons. (All units are in GeV.) The notation is as in Table I.

$H(n^jL_J)$	$m_{\text{expt.}}$	E^0	$E^{\text{phys.}}$	ϕ (%)
$B(1^{1/2}S_0)$	5.279	5.288	5.279	
$B(1^{1/2}S_1)$	5.325	5.288	5.324	
$B(1^{3/2}P_1)$		5.646	5.700	−6.00
$B(1^{1/2}P_0)$		5.675	5.706	
$B(1^{3/2}P_2)$		5.646	5.714	
$B(1^{1/2}P_1)$		5.675	5.742	6.00
$B(2^{1/2}S_0)$		5.840	5.886	
$B(2^{1/2}S_1)$		5.840	5.920	0.69
$B(1^{5/2}D_2)$		5.897	5.985	−1.96
$B(1^{5/2}D_3)$		5.897	5.993	
$B(1^{3/2}D_1)$		5.946	6.025	−0.69
$B(1^{3/2}D_2)$		5.946	6.037	1.96
$B(2^{1/2}P_0)$		6.076	6.163	
$B(2^{3/2}P_1)$		6.072	6.175	−9.11
$B(2^{3/2}P_2)$		6.072	6.188	0.50
$B(2^{1/2}P_1)$		6.076	6.194	9.11
$B(1^{7/2}F_3)$		6.102	6.220	−0.99
$B(1^{7/2}F_4)$		6.102	6.226	
$B(1^{5/2}F_2)$		6.153	6.264	−0.50
$B(1^{5/2}F_3)$		6.153	6.271	0.99
$B(3^{1/2}S_0)$		6.216	6.320	
$B(3^{1/2}S_1)$		6.216	6.347	

TABLE IV. Tabulated spectrum for B_s mesons. (All units are in GeV.) The notation is as in Table I.

$H(n^jL_J)$	$m_{\text{expt.}}$	E^0	$E^{\text{phys.}}$	ϕ (%)
$B_s(1^{1/2}S_0)$	5.369	5.381	5.373	
$B_s(1^{1/2}S_1)$	5.417	5.381	5.421	
$B_s(1^{1/2}P_0)$		5.767	5.804	
$B_s(1^{3/2}P_1)$		5.746	5.805	−7.19
$B_s(1^{3/2}P_2)$		5.746	5.820	
$B_s(1^{1/2}P_1)$		5.767	5.842	7.19
$B_s(2^{1/2}S_0)$		5.933	5.985	
$B_s(2^{1/2}S_1)$		5.933	6.019	0.64
$B_s(1^{5/2}D_2)$		5.999	6.095	−2.31
$B_s(1^{5/2}D_3)$		5.999	6.103	
$B_s(1^{3/2}D_1)$		6.041	6.127	−0.64
$B_s(1^{3/2}D_2)$		6.041	6.140	2.31
$B_s(2^{1/2}P_0)$		6.170	6.264	
$B_s(2^{3/2}P_1)$		6.168	6.278	−9.81
$B_s(2^{3/2}P_2)$		6.168	6.292	0.51
$B_s(2^{1/2}P_1)$		6.170	6.296	9.81
$B_s(1^{7/2}F_3)$		6.205	6.332	−1.15
$B_s(1^{7/2}F_4)$		6.205	6.337	
$B_s(1^{5/2}F_2)$		6.250	6.369	−0.51
$B_s(1^{5/2}F_3)$		6.250	6.376	1.15
$B_s(3^{1/2}S_0)$		6.310	6.421	
$B_s(3^{1/2}S_1)$		6.310	6.449	

We remark that m_u and m_s are mass parameters that differ from the constuent quark masses for an overall undetermined constant shift.

As a consistency check of our results we observe that the mass splitting $m_s - m_u \simeq 140$ MeV comes out in agreement with naive expectations based on Gell-Mann-Okubo type relations. This difference also agrees, within 1%, with the corresponding splitting determined in Ref. [6] and used in Ref. [5] as input for their calculations.

Remarkably $m_b \simeq M_b$ with much better agreement than expected. Moreover, $\lambda^{-1} \simeq 0.06$ fm is smaller than any other length scale involved in the problem, as was required.

Figure 2 shows some of the computed radial wave functions for non-strange mesons, $f^0_{n,l,j}(r)$ and $f^1_{n,l,j}(r)$. These wave functions do not include $1/m_h$ corrections, therefore they are the same for D and B mesons. The corresponding wave functions for strange mesons are very similar.

Figure 3 shows density plots for each couple of independent spin components of some of the computed light-quark wave functions, $|f^0_{n,l,j} Y^l_0|^2$ in black and $|f^1_{n,l,j} Y^{2j-l}_0|^2$ in gray. They represent the analogous, in the heavy meson systems, of the orbitals of the hydrogen atom.

F. Comparison with experiment

The comparison of our results to the present experimental information on the excitation spectrum of the (D, D_s, B, B_s) mesons is given in Table V. States which were used in the determining our best fit parameters are so indicated.

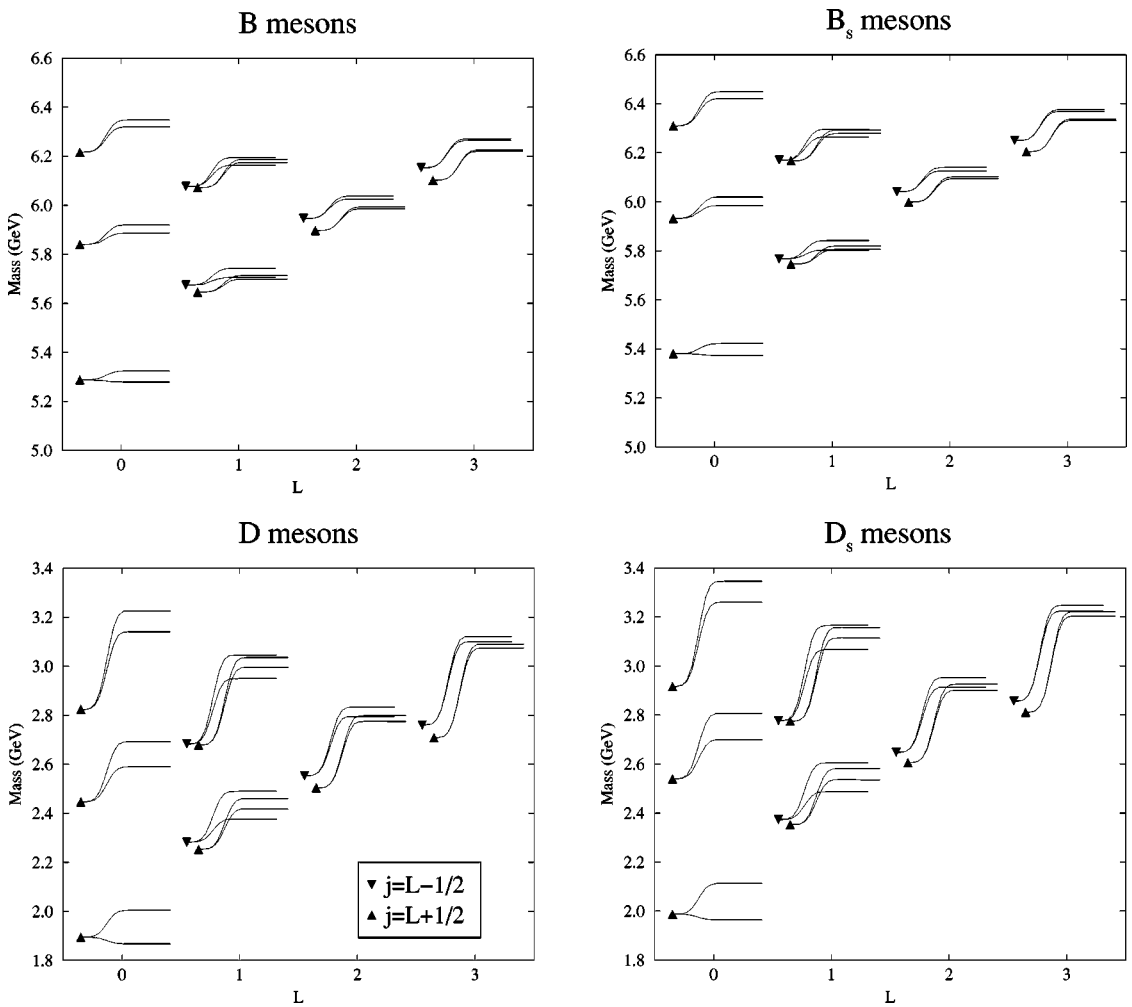

FIG. 1. Computed spectrum of excited states in the $\{D,D_s,B,B_s\}$ family. The plot shows the spectrum before and after $1/m_h$ corrections (including mixing). These corrections are responsible for the hyperfine splitting. The horizontal axis is the orbital angular momentum of the meson (l). For each value of l and j there is a doublet of states ($J=j-\frac{1}{2}$ and $J=j+\frac{1}{2}$, with lower and higher energy, respectively).

Our model is in excellent agreement with the better established P waves in the D and D_s systems. In particular the D_1^* $(1^{1/2}P_1)$ fits the recent measurement of CLEO [14]. For the P-waves of the B meson systems the agreement with preliminary measurements is somewhat less impressive.

However, many of the existing experimental fits for individual masses of these states relied on patterns of masses for the $j_l=1/2$ states not found in our model. For example, our relativistic quark model predict $m_{1,1,1/2,1}>m_{1,1,3/2,1}$ for a relatively broad range of parameters consistent with light spectroscopy.[1] Also, we obtain a splitting for the $^{1/2}P_J$ states more than twice as big as the splitting for the $^{3/2}P_J$ states.

Preliminary results from $L3$ [12] for the masses of P-wave B meson excitations are

$$B_1^*: m_{1,1,1/2,1}=(5.670\pm0.010_{\rm stat}\pm0.013_{\rm syst})\ \ {\rm GeV}\quad (L3) \tag{24}$$

$$B_2^*: m_{1,1,3/2,2}=(5.768\pm0.005_{\rm stat}\pm0.006_{\rm syst})\ \ {\rm GeV}\quad (L3). \tag{25}$$

The $L3$ results were derived using the constraint that $m_{1,1,1/2,1}-m_{1,1,1/2,0}=m_{1,1,3/2,2}-m_{1,1,3/2,1}=12$ MeV. This assumption is not realized in our model. Similar assumptions are needed for the extractions the masses of P-wave B meson excitations from OPAL [13] and CDF [18]. It would be interesting to reanalyze results using the pattern expected in this relativistic quark model. Finally, the observation of the D'^* by DELPHI [15] is not consistent with searches by CLEO [16] and OPAL [17].

G. Regge trajectories

We find that $E_{n,l,j,J}-M_h$ all lie on Regge trajectories parameterized by $mJ+q_{n,l,j}$ with $m\simeq0.7$ (as shown in Fig. 4). This is a well understood phenomenon for light spectroscopy

[1]This result is known in the literature as spin-orbit inversion. It was first predicted by Schnitzer [20] and later by the models of Isgur [21] and Ebert [22].

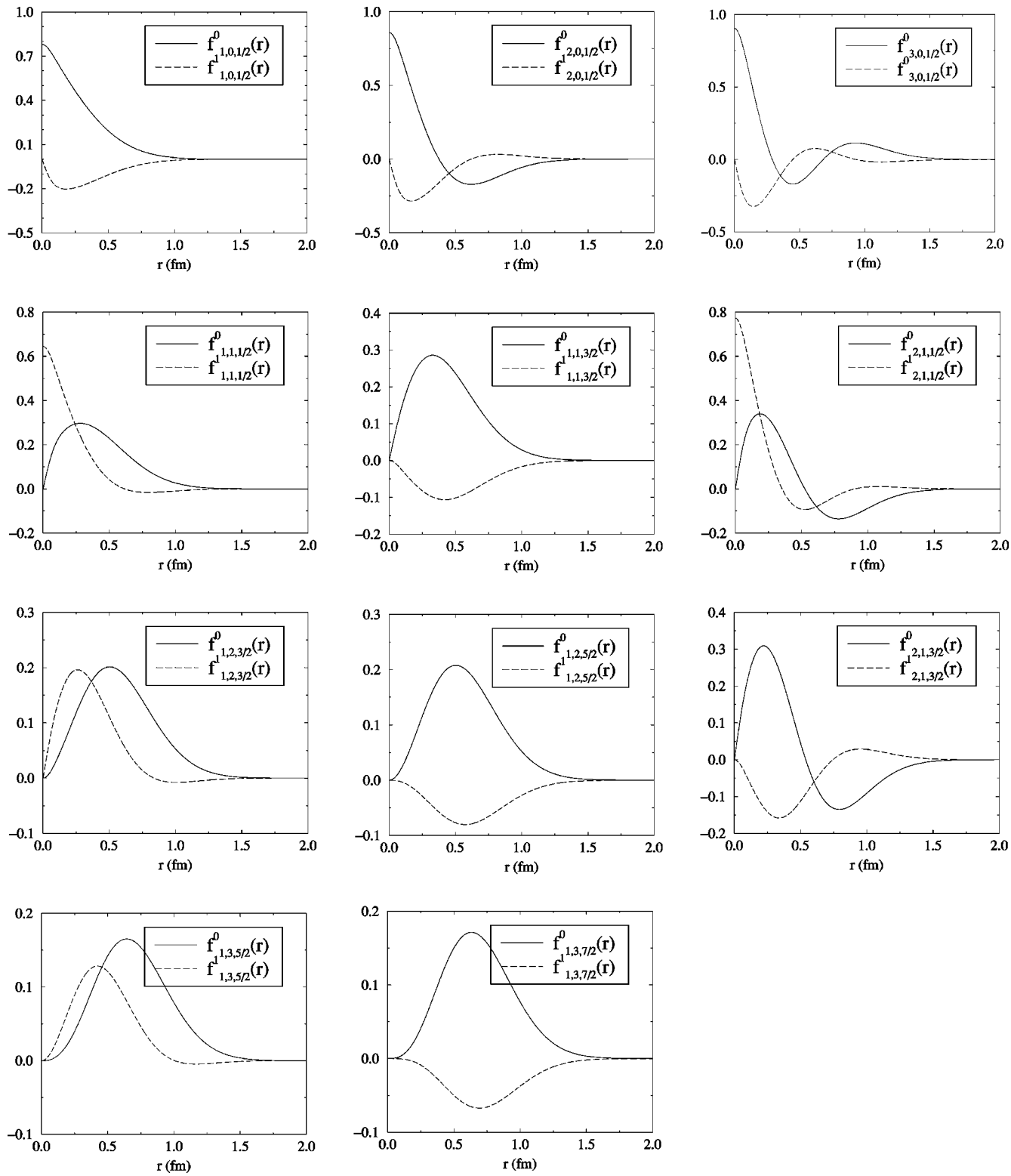

FIG. 2. Radial wave functions for some excited states (for non-strange mesons). The continuum (dashed) line refers to the $f^0(r)$ ($f^1(r)$) function. These plots do not include the mixing contribution.

but, for light mesons, $m \simeq 2\pi b$ (where b is the string tension). This is about a factor two bigger than we find. Our result can be explained in the non-relativistic limit or, alternatively, in a simple and naive string picture: Ignoring the short distance behavior of the potential, we model the meson as a classical light quark attached through a string to the center of mass of the system. The energy of the system, E, (i.e., the energy of the string) is related to the classical an-

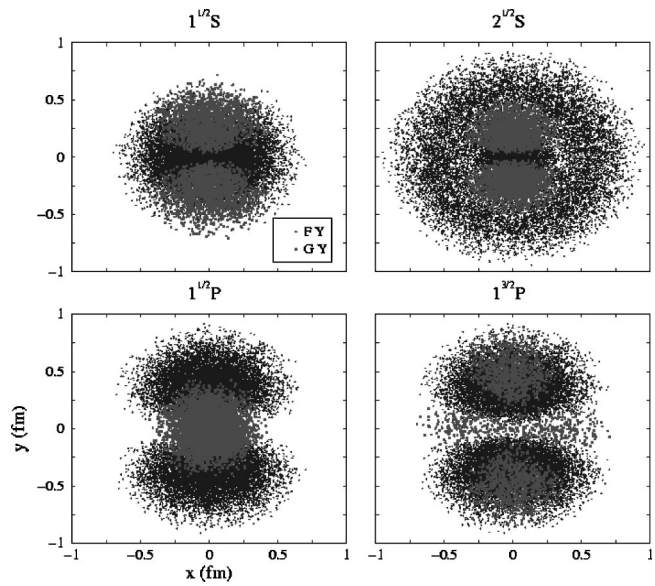

FIG. 3. Orbitals for some excited B mesons.

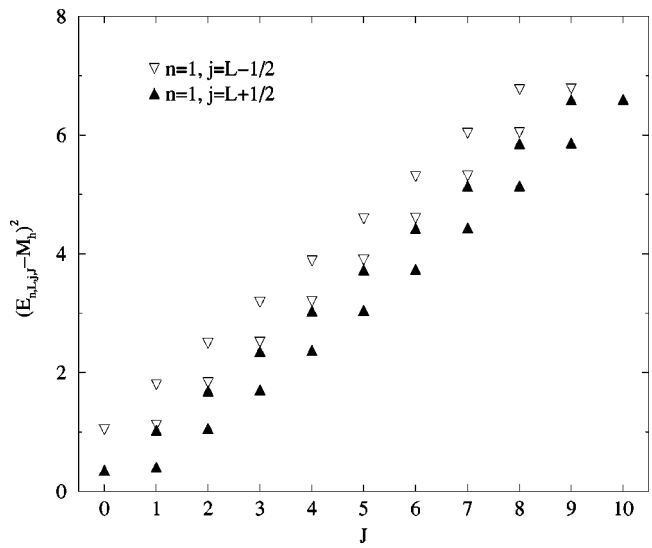

FIG. 4. Regge trajectories for some of our orbitally excited heavy mesons.

gular momentum, J, by $E^2=\pi bJ$. The factor two in the light meson picture can be explained with the fact that the latter rotate around a center of mass that is located at the middle of the string while for heavy-light mesons the center of mass coincides with one of the two ends of the string.

This simple picture shows that the bulk of an heavy meson mass is dominated by the mass of the heavy quark plus the potential energy associated to the large distance interaction ($\propto br$), and again gives support to our assumption that short distance behavior of the potential has a small contribution to the spectrum.

TABLE V. The heavy-light spectrum compared to experiment. We report the difference between the excited state masses and the ground state (D or B) in each case.

Charmed meson masses (MeV)				Bottom meson masses (MeV)			
	Model	Expt.	[Ref.]		Model	Expt.	[Ref.]
D^*-D	137 [a]	141,142	[11]	B^*-B	45 [a]	46	[11]
D_0^*-D	512			B_0^*-B	427		
D_1^*-D	622	596(53)	[14]	B_1^*-B	463	391(16)	[12] [b]
D_1-D	549 [a]	558(2)	[11]	B_1-B	421	459(9)	[13] [b]
				B_1-B	421	431(20)	[18] [b]
D_2^*-D	592 [a]	594(2)	[11]	B_2^*-B	435	489(8)	[12] [b]
						460(13)	[19] [b]
				$(B^{**}-B)$		418(9)	[11] [c]
$D'-D$	721			$B'-B$	607		
D'^*-D	824	772(6)	[15]	B'^*-B	641		
		not seen	[16,17]				
D_s-D	97 [a]	99,104	[11]	B_s-B	94 [a]	90(2)	[11]
$D_s^*-D_s$	148 [a]	144	[11]	$B_s^*-B_s$	48 [a]	46	[11]
$D_{s0}^*-D_s$	512			$B_{s0}^*-B_s$	431		
$D_{s1}^*-D_s$	640			$B_{s1}^*-B_s$	469		
$D_{s1}-D_s$	570 [a]	566(1)	[11]	$B_{s1}-B_s$	432		
$D_{s2}^*-D_s$	616 [a]	605(2)	[11]	$B_{s2}^*-B_s$	447		
				$(B_s^{**}-B_s)$		484(15)	[11] [c]
$D_s'-D_s$	735			$B_s'-B_s$	612		
$D_s'^*-D_s$	841			$B_s'^*-B_s$	646		

[a]Experimental input to model parameters fit.
[b]Theoretical estimates for some of the mass splittings have been used as input.
[c]Experimental signal is a sum over resonances with $J=0,1,2$.

III. HADRONIC TRANSITIONS

A. Transition amplitudes

We start by considering the most general hadronic transition of the form

$$H' \to H + x \tag{26}$$

where H' and H are two heavy-light mesons containing the same heavy quark, with wavefunctions $\Psi_{n',l',j',J',M'}$ and $\Psi_{n,l,j,J,M}$, respectively, and x can be any light meson with momentum $\mathbf{p}$. Although we keep our formalism general, in this paper we only compute numerically decays in which x is a pseudoscalar meson belonging to the flavor octet (π, K, η).

In the context of the chiral quark model [4] this transition is mediated by an effective interaction of the form

$$\mathcal{L}_{\rm int} = \frac{g_A^8}{\sqrt{2} f_x} \overline{q}_i' X \mathcal{M}^{ij} q_j + \mathcal{O}(\partial^2) \tag{27}$$

where f_x can be identified with $f_\pi \simeq 130$ MeV and g_A^8 is an effective coupling. $X = \not{p}\gamma^5$ is the spin structure associated to the transition, i,j are $\mathrm{SU}(3)_{\rm flavor}$ indices and

$$\mathcal{M} = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \tag{28}$$

is the usual $SU(3)_{L+R}$ invariant representation of the pseudoscalar mesons.

The transition mediated by this Lagrangian is associated with the following matrix element:

$$I^{H'Hx}(\mathbf{p}) = \frac{i g_A^8 \zeta}{\sqrt{2} f_x} \int \overline{\Psi}_{n,l,j,J,M}(z) X e^{i\mathbf{p}\cdot\mathbf{z}} \Psi_{n',l',j',J',M'}(z) d^3 z \tag{29}$$

where ζ is a coefficient that characterize the flavor structure of the decay. A list of all possible cases has been derived from Eq. (28) and is reported in Table VI. The physical η and η' are of course mixtures of the ideal η_8 and η_1. In particular, $\eta = \eta_8 \cos(\theta_p) - \eta_1 \sin(\theta_p)$ where $\theta_p \approx -10.1°$ with a large uncertainty. In this work we ignore this mixing and we assume $\theta_p = 0$. The corrective multiplicative factor for a different choice for θ_p can be derived by the reader using Table VI.

The exponential in Eq. (29) can be expanded in products of spherical harmonics and spherical Bessel functions, thus giving

TABLE VI. List of decay channels for B (or D) mesons with the corresponding flavor factor ζ.

$H' \to H + x$	ζ	$H' \to H + x$	ζ
$B^0 \to B^0 + \pi^0$	1	$B_s \to B_s + \eta_8$	$-2/\sqrt{3}$
$B^\pm \to B^\pm + \pi^0$	1	$B_s \to B^0 + K$	$\sqrt{2}$
$B^\pm \to B^0 + \pi^\pm$	$\sqrt{2}$	$B_s \to B^\pm + K^\mp$	$\sqrt{2}$
$B^0 \to B^\pm + \pi^\mp$	$\sqrt{2}$	$B^0 \to B_s + \overline{K}$	$\sqrt{2}$
$B^0 \to B^0 + \eta_8$	$1/\sqrt{3}$	$B^\pm \to B_s + K^\pm$	$\sqrt{2}$
$B^\pm \to B^\pm + \eta_8$	$1/\sqrt{3}$	$B_q \to B_q + \eta_1$	$\sqrt{\frac{2}{3}} + \mathcal{O}\left(\frac{1}{N_c}\right)$

$$I^{H'Hx}(\mathbf{p}) = \sum_{l_x, m_x} Y_{m_x}^{l_x *}(\hat{p}) C_{J,M;l_x,m_x}^{J',M'} \mathcal{A}_{l_x}^{H'Hx}(X,p). \tag{30}$$

Equation (30) implicitly defines the transition amplitude, $\mathcal{A}_{l_x}^{H' \to Hx}(X,p)$, for a x in a given eigenstate l_x of its angular momentum. By projecting the matrix X on the basis presented in the Appendix A, the transition amplitude can be rewritten as a linear combination of terms, each factorized into a radial part and a spin dependent part

$$\mathcal{A}_{l_x}^{H'Hx}(X,p) = \frac{i g_A^8 \zeta}{\sqrt{2} f_x} \sum_{ab=\{0,1\}} \sum_k c_{l_x}^{ab,k}(X) \times \int_0^\infty f_{n',l',j'}^a(r) j_k(rp) f_{n,l,j}^b(r) r^2 dr. \tag{31}$$

The coefficients $c_{l_x}^{ab,k}(X)$ depend on the quantum numbers of the mother and the daughter heavy mesons. Their explicit expression is given in the Appendix A. The integrals are computed numerically.

One can extend our analysis for octet pseudoscalar transitions to the approximately flavor singlet η'. In the large N_c limit the η_1 combines with the octet to form a nonet. In that case the effective interaction in the Lagrangian takes the form

$$\mathcal{L}_{\rm int} = \frac{g_A^1 \zeta}{\sqrt{2} f_{\eta_1}} \overline{q}_i' X \eta_1 q_i + \mathcal{O}(\partial^2) \tag{32}$$

with $X = \not{p}$ and $g_A^1 \simeq g_A^8$. This symmetry is badly broken in QCD. However, it is reasonable to assume that in these transitions that the form [Eq. (32)] still holds. If one further assumes that the spatial wave functions of π and η' are approximately equal, one obtains $f_{\eta'} \simeq f_\pi$. Hence the coefficient ζ can be set to $\sqrt{2/3}$ both for heavy strange and nonstrange decaying heavy mesons.

The situation for decays in which x is a light vector mesons (ρ, ω, K^*) is different. When compared to the pseudoscalar mesons, they have a different spin coupling to the quarks ($X = \not{\epsilon}$ where ϵ_μ is the polarization vector of the me-

TABLE VII. List of partial decay rates for $1S$ D mesons. The measured D^* and D masses [11] are used in these results.

Channel	l_π	p_π (MeV)	$\Gamma_\pi/(g_A^8)^2$(keV)
$D^{\star 0}(1^{1/2}S_1)\to D^0(1^{1/2}S_0)+\pi^0$	1	$42.8\pm.2$	62 ± 1
$D^{\star +}(1^{1/2}S_1)\to D^0(1^{1/2}S_0)+\pi^+$	1	$39.4\pm.5$	97 ± 3
$D^{\star +}(1^{1/2}S_1)\to D^+(1^{1/2}S_0)+\pi^0$	1	$38.1\pm.3$	44 ± 1

son), a different effective coupling ($g_V\neq g_A^8$), and a different wave function ($f_\rho\neq f_\pi$). With these replacements Eq. (31) remains valid for decays with emission of light vector mesons.[2] The detailed study of these vector meson transitions is deferred to a future paper.

B. Partial widths

The partial width for the transition in Eq. (26) (for a light meson x emitted with total momentum p and angular momentum l_x) is given by

$$\Gamma_x(H'\to H+x;\ \ l_x)=\frac{p}{8\pi^2}\frac{2J+1}{2J'+1}\frac{m_H}{m_{H'}}|\mathcal{A}_{l_x}^{H'Hx}(X,p)|^2 \tag{33}$$

where $m_{H'}$ and m_H are the masses of the mother and daughter heavy mesons respectively.

The total hadronic decay width (via a pseudoscalar meson transition) is defined simply as the sum of the partial widths:

$$\Gamma_{\mathcal{M}}^{H'}=\sum_H\ \sum_{x=\{\pi,\eta_8,K\}}\ \sum_{l_x}\ \Gamma_x(H'\to H+x;\ \ l_x). \tag{34}$$

Transitions involving excited states very near their kinematic threshold for an allowed decay (e.g. where the light pseudoscalar momentum is less than 100 MeV) are extremely sensitive to our calculated mass values. This is particularly true for the allowed transitions within the $1S$ multiplets. In these cases, even the small mass differences between the the charged and neutral states are important. Using the physical masses for the various D mesons [11], the individual pion transitions are shown in Table VII.

After removing these phase space uncertainties, reasonable variations in our model parameters gave variations of about 10% in the overall hadronic widths. The listed branching ratios with emission of a π are flavor blind and sum over the final state pion charge (i.e., they have been computed with $\zeta=\sqrt{3}$). Each exclusive decay can be deduced by correcting for this factor using Table VI to determine the relative strength of the charged and neutral decays. In addition, a small phase space correction should be included appropriate to the slight difference in the masses of the various charge

[2]It is possible to relate these the pseudoscalar and vector couplings and coefficients within the context of an approximate $SU(6)_W$ symmetry for the low-lying states.

TABLE VIII. The heavy-light $1P$ state hadronic transition rates for D and D_s mesons. $H'\to H+x$. Decays denoted with an (*) are allowed only because of the order $1/m_h$ mixing of states. p_x and $\Gamma_x/(g_A^8)^2$ are in MeV.

$H'(n'^{j'}l_{J'})$	$H(n^jl_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
$D(1^{1/2}P_0)$	$D(1^{1/2}S_0)$	π	0	437	189
$D(1^{3/2}P_1)$	$D(1^{1/2}S_1)$	π	0	355	(*) 1.7
	$D(1^{1/2}S_1)$	π	2	355	14.5
$D(1^{3/2}P_2)$	$D(1^{1/2}S_0)$	π	2	506	24.6
	$D(1^{1/2}S_1)$	π	2	394	13.7
$D(1^{1/2}P_1)$	$D(1^{1/2}S_1)$	π	0	420	181
$D_s(1^{1/2}P_0)$	$D(1^{1/2}S_0)$	K	0	325	236
$D_s(1^{3/2}P_1)$	$D(1^{1/2}S_1)$	K	0	175	(*) 1.89
	$D(1^{1/2}S_1)$	K	2	175	0.3
$D_s(1^{3/2}P_2)$	$D(1^{1/2}S_0)$	K	2	442	8.9
	$D(1^{1/2}S_1)$	K	2	264	1.4
	$D_s(1^{1/2}S_0)$	η	2	248	0.4
$D_s(1^{1/2}P_1)$	$D(1^{1/2}S_1)$	K	0	302	224

states. These rates are shown in Table VIII for the D and D_s mesons and in Table IX for the B and B_s mesons. For the $1^{3/2}P_{(1,2)}$ B_s states, our model predicts that they are below threshold for K transitions to the the $1^{1/2}S_{(0,1)}$ B states. However, this is very sensitive to the details of the model. So, for completeness, we note the partial rates divided by the appropriate phase space factor at $p_K=0$ in Table X. A list of the allowed transitions for other low-lying excited states is reported in Appendix B.

C. Comparison to lattice results

As one more consistency check of our model we compare our prediction for the transition[3] $B\to B^*+\pi$ with model independent results coming from lattice simulations. We define

$$\mathcal{A}^{BB^*\pi}(r)=\frac{1}{3}\sum_{\mu=1,2,3}\int\langle B^*|A_\mu(\mathbf{r})|B\rangle \mathrm{d}\Omega_r \tag{35}$$

with $A_\mu(\mathbf{r})=\overline{q}(t,\mathbf{r})\gamma_\mu\gamma^5 q(t,\mathbf{r})$ at $t=0$. The same matrix element can be expressed in terms of our radial wave function and, in the limit $p_\pi\to 0$,

$$\mathcal{A}^{BB^*\pi}(r)=-g_A^8\left[(f^0_{1,0,1/2}(r))^2-\frac{1}{3}(f^1_{1,0,1/2}(r))^2\right]+\mathcal{O}(p_\pi). \tag{36}$$

[3]Even if this transition is kinematically forbidden it is physically relevant to the $B\to\pi+l+\overline{\nu}$ exclusive decay under the assumption of vector meson dominance.

TABLE IX. The $1P$ state hadronic transition rates for B and B_s systems. $H' \to H + x$. Decays denoted with an (*) are allowed only because of the order $1/m_h$ mixing of states. Values for p_x and $\Gamma_x/(g_A^8)^2$ are in MeV.

$H'(n'^{j'}l_{J'})$	$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
$B(1^{1/2}P_0)$	$B(1^{1/2}S_0)$	π	0	388	186
$B(1^{3/2}P_1)$	$B(1^{1/2}S_1)$	π	0	338	(*) 0.5
	$B(1^{1/2}S_1)$	π	2	338	13.1
$B(1^{3/2}P_2)$	$B(1^{1/2}S_0)$	π	2	396	10.6
	$B(1^{1/2}S_1)$	π	2	352	9.5
$B(1^{1/2}P_1)$	$B(1^{1/2}S_1)$	π	0	381	180
$B_s(1^{1/2}P_0)$	$B(1^{1/2}S_0)$	K	0	170	159
$B_s(1^{1/2}P_1)$	$B(1^{1/2}S_1)$	K	0	153	143

g_A^8 is the effective coupling of the transition as defined in Eq. (27).[4] In the chiral quark model g_A^8 is an effective parameter and it is has to be given as input. On the other side, in the context of lattice computations, the matrix element in Eq. (35) follows directly from first principles (the QCD Lagrangian) and can be computed explicitly. By fitting the lattice results of Ref. [24] with our prediction for $\mathcal{A}^{BB^*\pi}(r)$ as function of g_A^8 we are able to determine[5]

$$g_A^8 = Z_A g_A^{8\,(\mathrm{lattice})} = 0.53 \pm 0.11 \tag{37}$$

where $g_A^{8\,(\mathrm{lattice})}$ is the naive lattice result extracted from the fit and $Z_A = 0.78$ is the lattice matching factor discussed in Ref. [24]. The error includes the statistical error due to the simulation and the fits ($\simeq 10\%$), and an estimate of the systematic error in the matching coefficient and in the the chiral extrapolation. This is a preliminary result, as those of Ref. [24] are, because of the small lattice size and the poor chiral extrapolation. In any case our result is in agreement with the lattice determination of Ref. [25], $g_A^8 = 0.61 \pm 0.13$, and with experimental results from nucleon and hyperon β decays, $g_A^8 = 0.58 \pm 0.02$ [26]. A more precise lattice determination is possible and may be carried out in the near future.

Apart for the overall normalization given by g_A^8, the radial dependence of the function $\mathcal{A}_{B^*B\pi}(r)$ is predicted independently by our model and by the lattice computation. In Fig. 5 we present a comparison between our analytical result, with adjusted normalization, and the lattice data. We believe this comparison provides a satisfactory consistency check of the two methods.[6]

[4]Note from Eq. (36) that in the non-relativistic limit the coupling g_A^8 coincides with the coupling constant g that appears in the heavy meson chiral Lagrangian [23]. The lattice result for this quantity is $g = 0.42 \pm 0.09$.

[5]We set to zero terms of the order $\mathcal{O}(p_\pi)$ for consistency with the lattice computation.

TABLE X. Decay rates for $1^{3/2}P_{(1,2)}$ B_s mesons with phase space dependence divided out. These states are very near the kinematical threshold in our model. Values for p_x and $\Gamma_x/(g_A^8)^2$ are in MeV.

$H'(n'^{j'}l_{J'})$	$H(n^j l_J)$	x	l_x	$\Gamma_x/(g_A^8)^2 \times (100/p_x)^{(2l_x+1)}$
$B_s(1^{3/2}P_1)$	$B(1^{1/2}S_1)$	K	0	(*) 4.92×10^{-1}
$B_s(1^{3/2}P_1)$	$B(1^{1/2}S_1)$	K	2	2.38×10^{-2}
$B_s(1^{3/2}P_2)$	$B(1^{1/2}S_0)$	K	2	9.48×10^{-3}
$B_s(1^{3/2}P_2)$	$B(1^{1/2}S_1)$	K	2	1.43×10^{-2}

D. Comparison to experiment

The total width of the D^{*+} meson has recently been measured by the CLEO Collaboration [27]. They obtain $\Gamma = 96 \pm 4_{\mathrm{stat}} \pm 22_{\mathrm{syst}}$(keV). Combining this measurement with the well-known branching ratios for the various pionic transitions gives a measurement of chiral coupling constant. We obtain

$$g_A^8 = 0.82 \pm 0.09 \tag{38}$$

within our model.

Within the chiral quark model, the coupling determined from any transition should agree (i.e. the coupling is independent of particular initial or final heavy-light states). Table XI lists the various determinations of g_A^8 from existing data. Within the existing large uncertainties the various determinations are consistent.

IV. CONCLUSIONS

We have computed the spectrum and hadronic decay width of the excited D, D_s, B and B_s mesons using a relativistic quark model for the masses and wave function of the heavy-light mesons. This work is based on that of Refs. [8,6,5] but departs from these previous works because we choose a simpler form for the potential and determined its parameters exclusively from fitting the experimental heavy-light spectrum. Moreover, we computed all corrections within the model to order $1/m_h$, including mixing between nearby states with the same J^P.

Our spectrum results agree very well with the existing data in the D and D_s systems. The agreement in the B and B_s systems is also fairly good but the experimental situation for the P states is not yet completely resolved.

For example, our model predicts a spin-orbit inversion for the excited P-wave states in these systems. This agrees with the recent CLEO [14] results for the D mesons; but is in

[6]It also suggests a possible use of chiral quark model wave functions to isolate excited states contributions in lattice correlation functions.

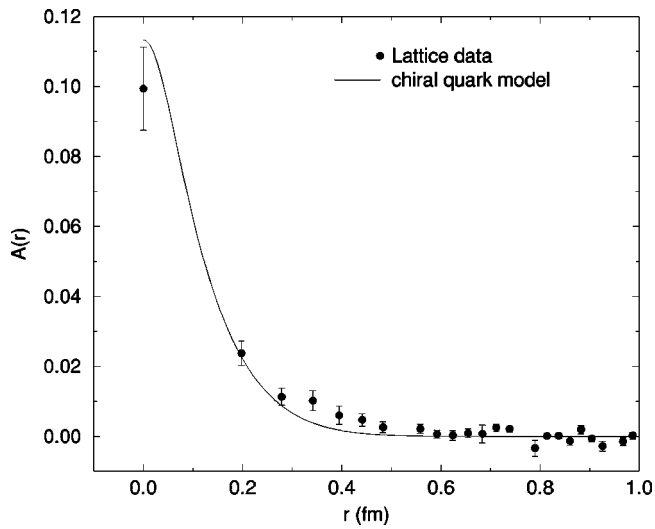

FIG. 5. Comparison between a prediction of our model and the lattice QCD result [24].

disagreement with preliminary $L3$ results [12] for the B mesons. However, in our model we find that the splitting between the $j_l=1/2$ P-wave states ($J=0,1$) is more than twice as large as the splitting of the $j_l=3/2$ P-wave states ($J=1,2$), which is inconsistent with the assumption of equal splitting used by $L3$ [12] in their analysis.

The information on the spectrum and the wave function was used to compute a complete list of allowed hadronic decays for these excited mesons into lower energy states with emission of a light pseudoscalar meson (η, π or K) and their relative branching ratios.

We also compared our model prediction for $\langle B^*|A^*_\mu(r)|B\rangle$ with lattice results. We find good agreement between the shapes of the decay amplitudes. We used this comparison to extract a preliminary lattice determination of g_A^8, the effective coupling of the chiral quark model. We find the value $g_A^8=0.53\pm0.11$. Using this value for g_A^8 to translate the total hadronic decay widths in physical units, we would conclude that the widths of the D meson P waves are consistently below experimental findings. Of course, the lattice results have not yet been extrapolated to the continuum.

Comparison of our results with a recent experiment measurement of the D^{*+} width [27] yields $g_A^8=0.82\pm0.09$. Using this value, we find much better overall agreement with experimental findings.

Finally, there is also a whole set of hadronic decays with emission of light vector mesons that we did not include in our study. For the higher excited states these transitions give an important contribution to the total physical widths. We intend to study these transitions in a future paper.

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APPENDIX A

In order to be able to factorize the expression for the transition amplitude, Eq. (30), into radial and angular parts, it is convenient to adopt the following basis for the Γ matrices:

$$\Gamma^{00}_\mu=\begin{pmatrix}\sigma_\mu & 0\\ 0 & 0\end{pmatrix},\quad \Gamma^{01}_\mu=\begin{pmatrix}0 & \sigma_\mu\\ 0 & 0\end{pmatrix}, \tag{A1}$$

TABLE XI. The $1S$ and $1P$ heavy-light hadronic transition rates compared to experiment. All widths are in MeV unless otherwise indicated. Experimental values are from the Particle Data Group (PDG) [11] unless otherwise indicated.

State	Width (expt.)	[Ref.]	$\Gamma/(g_A^8)^2$ (model)	g_A^8
$D^+(1^{1/2}S_1)$	$96\pm4\pm22$ (keV)	[27]	143 (keV) [a]	0.82 ± 0.09
$D^0(1^{3/2}P_1)$	$18.9^{+4.6}_{-3.5}$		16	$1.09^{+0.12}_{-0.11}$
$D^+(1^{3/2}P_1)$	28 ± 8		16	$1.32^{+0.18}_{-0.27}$
$D^+(1^{1/2}P_1)$	$290^{+101}_{-79}\pm26\pm36$	[14]	181	1.27 ± 0.22
$D^0(1^{3/2}P_2)$	23 ± 5		38	0.77 ± 0.08
$D^+(1^{3/2}P_2)$	25 ± 8		38	$0.81^{+.012}_{-0.14}$
$D_s(1^{3/2}P_1)$	$\leqslant2.3$		2.0	$\leqslant1.07$
$D_s(1^{3/2}P_2)$	15 ± 5		10.9	$1.17^{+0.18}_{-0.11}$
$B(1^{1/2}P_1)$	73 ± 44	[12] [b]	180	$0.64^{+0.17}_{-0.21}$
$B(1^{3/2}P_1)$	$18^{+15}_{-13}{}^{+29}_{-23}$	[13] [b]	14.0	$1.13^{+0.77}_{-1.13}$
$B(1^{3/2}P_2)$	41 ± 43	[12] [b]	20.0	$1.43^{+0.61}_{-1.43}$

[a]Theoretical value corrected for phase space observed mass (see Table VII) and the 1.6% branching ratio to $D^++\gamma$ [11].

[b]Experimental results depend strongly on model dependent assumptions.

$$\Gamma_\mu^{10}=\begin{pmatrix} 0 & 0 \\ \sigma_\mu & 0 \end{pmatrix}, \quad \Gamma_\mu^{11}=\begin{pmatrix} 0 & 0 \\ 0 & \sigma_\mu \end{pmatrix} \tag{A2}$$

where σ_0 is the 2×2 identity and σ_i are the usual Pauli matrices. On this basis we obtain simple expressions for the $c_{l_x}^{ab,k}(X)$ coefficients[7]

$$c_{l_x}^{00,k}(X)=c_0\mathrm{tr}(\gamma^0 X\Gamma_\mu^{00})\langle j',m',l'|\sigma^\mu Y_{k,m_k}|j,m,l\rangle \tag{A4}$$

$$c_{l_x}^{01,k}(X)=c_0\mathrm{tr}(\gamma^0 X\Gamma_\mu^{01})\langle j',m',l'|\sigma^\mu Y_{k,m_k}|j,m,2j-l\rangle \tag{A5}$$

$$c_{l_x}^{10,k}(X)=c_0\mathrm{tr}(\gamma^0 X\Gamma_\mu^{10})\langle j',m',2j'-l'|\sigma^\mu Y_{k,m_k}|j,m,l\rangle \tag{A6}$$

$$c_{l_x}^{11,k}(X)=c_0\mathrm{tr}(\gamma^0 X\Gamma_\mu^{11})\langle j',m',2j'-l'|\sigma^\mu Y_{k,m_k}|j,m,2j-l\rangle \tag{A7}$$

where

$$c_0=4\pi(-i)^{l_x}\frac{\sum_s C_{j',M'-s;1/2,s}^{J',M'}C_{j,M-s;1/2,s}^{J,M}}{C_{l_x,m_x;j,M}^{J',M'}}\frac{1}{2}. \tag{A8}$$

For the particular case $X=\not{p}\gamma^5$ the angular dependence from the **p** vector disappears and we obtain the following explicit expression:

$$c_{l_x}^{00,k}(\not{p}\gamma^5)=+c_2|p|\langle j',l'||T_{l_x}^{(k)}||j,l\rangle \tag{A9}$$

$$c_{l_x}^{01,k}(\not{p}\gamma^5)=+c_1p_0\langle j',l'||Y_k||j,2j-l\rangle \tag{A10}$$

$$c_{l_x}^{10,k}(\not{p}\gamma^5)=-c_1p_0\langle j',2j'-l'||Y_k||j,l\rangle \tag{A11}$$

$$c_{l_x}^{11,k}(\not{p}\gamma^5)=-c_2|p|\langle j',2j'-l'||T_{l_x}^{(k)}||j,2j-l\rangle \tag{A12}$$

with

$$c_1=4\pi i(-i)^{l_x}(-)^{j'-(1/2)+J'+l'-l+l_x}[J]\times\begin{Bmatrix} l_x & j & j' \\ \frac{1}{2} & J' & J \end{Bmatrix}\delta_{k,l_x} \tag{A13}$$

$$c_2=4\pi(-i)^k(-)^{j'-(1/2)+J'-k}[Jk]\times\begin{Bmatrix} l_x & j & j' \\ \frac{1}{2} & J' & J \end{Bmatrix}\begin{pmatrix} 1 & k & l_\pi \\ 0 & 0 & 0 \end{pmatrix}. \tag{A14}$$

Explicit expressions for the Wigner-Ekkart reduced tensors are

$$\langle l'j'\|Y_k\|lj\rangle=\frac{[jj'll'k]}{\sqrt{4\pi}}(-)^{j+l'-l+1/2}\begin{pmatrix} l' & k & l \\ 0 & 0 & 0 \end{pmatrix}\times\begin{Bmatrix} k & l' & l \\ \frac{1}{2} & j & j' \end{Bmatrix} \tag{A15}$$

and

$$\langle l'j'\|T_{l_x}^{(k)}\|lj\rangle=\sqrt{\frac{3}{2\pi}}[jj'll'kl_x](-)^{l'}\begin{pmatrix} l' & k & l \\ 0 & 0 & 0 \end{pmatrix}\times\begin{Bmatrix} \frac{1}{2} & l' & j' \\ \frac{1}{2} & l & j \\ 1 & k & l_x \end{Bmatrix}. \tag{A16}$$

Our expression for the spin structure of the decay amplitudes, reported in this appendix, disagree with Ref. [5] in the overall phase factor. This factor does not affect their results but is relevant in case of mixing.

APPENDIX B

In this appendix we list the hadronic transitions $H'\to H+x$ for the low-lying excited states not discussed in Sec. III B. Table XII lists the transitions in decending order according to (i) the flavor of the decaying heavy meson H'; (ii) its mass; (iii) the decay channel.

Transitions with a branching ratio less than 1% are not reported. Some decays are marked with an asterisk. These are decays that apparently do not conserve the heavy quark spin ($|j'-j|\leqslant l_x\leqslant|j'+j|$) but, we remind the reader, our initial and final states are physical states and therefore such decays become allowed because of mixing effects.

[7]In this appendix we follow the notation of Elbaz [28] where

$$[a_1\ldots a_n]=\sqrt{(2a_1+1)\ldots(2a_n+1)}. \tag{A3}$$

TABLE XII. Hadronic transitions $H' \to H + x$ for the low-lying states not discussed in Sec. III B.

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(2^{1/2}S_0)$	$m = 2.589$ GeV		
$D(1^{1/2}S_1)$	π	1	504	14.5
$D(1^{1/2}P_0)$	π	0	154	7.0

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(2^{1/2}S_1)$	$m = 2.692$ GeV		
$D(1^{1/2}S_0)$	η	1	518	1.4
$D(1^{1/2}S_0)$	π	1	688	39.9
$D(1^{1/2}S_1)$	π	1	587	30.4
$D(1^{3/2}P_1)$	π	2	225	1.9
$D(1^{1/2}P_1)$	π	0	141	6.2
$D_s(1^{1/2}S_0)$	K	1	460	5.6

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(1^{5/2}D_2)$	$m = 2.775$ GeV		
$D(1^{1/2}S_1)$	π	3	652	20.1
$D(1^{1/2}P_0)$	π	2	347	7.3
$D(1^{3/2}P_1)$	π	2	308	4.3
$D(1^{3/2}P_2)$	π	2	266	1.4
$D(1^{1/2}P_1)$	π	2	236	1.3
$D_s(1^{1/2}S_1)$	K	3	387	0.5

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(1^{3/2}D_1)$	$m = 2.795$ GeV		
$D(1^{1/2}S_0)$	η	1	620	4.0
$D(1^{1/2}S_0)$	π	1	764	18.6
$D(1^{1/2}S_1)$	π	1	668	6.8
$D(1^{3/2}P_1)$	π	0	328	87.2
$D(1^{3/2}P_1)$	π	2	328	2.4
$D(1^{3/2}P_2)$	π	2	286	1.4
$D_s(1^{1/2}S_0)$	K	1	566	15.0
$D_s(1^{1/2}S_1)$	K	1	412	3.3

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(1^{5/2}D_3)$	$m = 2.799$ GeV		
$D(1^{1/2}S_0)$	η	3	624	0.7
$D(1^{1/2}S_0)$	π	3	767	18.0
$D(1^{1/2}S_1)$	π	3	671	13.2
$D(1^{3/2}P_1)$	π	2	331	2.5
$D(1^{3/2}P_2)$	π	2	290	5.2
$D(1^{1/2}P_1)$	π	2	261	3.5
$D_s(1^{1/2}S_0)$	K	3	570	2.1

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(1^{3/2}D_2)$	$m = 2.833$ GeV		
$D(1^{1/2}S_1)$	η	1	528	4.4
$D(1^{1/2}S_1)$	π	1	697	22.9
$D(1^{1/2}P_0)$	π	2	400	1.5
$D(1^{3/2}P_1)$	π	2	363	3.6
$D(1^{3/2}P_2)$	π	0	323	87.0
$D(1^{3/2}P_2)$	π	2	323	3.2
$D_s(1^{1/2}S_1)$	K	1	456	13.0

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(2^{1/2}P_0)$	$m = 2.949$ GeV		
$D(1^{1/2}S_0)$	η	0	756	11.7
$D(1^{1/2}S_0)$	π	0	875	88.0
$D(1^{3/2}P_1)$	π	1	467	15.2
$D(1^{1/2}P_1)$	π	1	403	60.6
$D(2^{1/2}S_0)$	π	0	311	51.9
$D_s(1^{1/2}S_0)$	K	0	706	66.6

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(2^{3/2}P_1)$	$m = 2.995$ GeV		
$D(1^{1/2}S_1)$	η	2	685	2.7
$D(1^{1/2}S_1)$	π	2	818	62.5
$D(1^{1/2}P_0)$	π	1	540	1.5
$D(1^{3/2}P_1)$	π	1	506	5.9
$D(1^{3/2}P_2)$	π	3	470	4.0
$D(1^{1/2}P_1)$	π	1	444	3.9
$D(2^{1/2}S_1)$	π	2	255	5.1
$D(1^{3/2}D_1)$	π	0	138	3.8
$D_s(1^{1/2}S_1)$	K	2	620	9.9

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H' = D(2^{3/2}P_2)$	$m = 3.035$ GeV		
$D(1^{1/2}S_0)$	η	2	828	3.3
$D(1^{1/2}S_0)$	π	2	936	53.4
$D(1^{1/2}S_1)$	η	2	721	2.3
$D(1^{1/2}S_1)$	π	2	848	47.1
$D(1^{3/2}P_1)$	π	3	541	4.3
$D(1^{3/2}P_2)$	π	1	505	4.2
$D(1^{3/2}P_2)$	π	3	505	2.2
$D(1^{1/2}P_1)$	π	1	480	6.1
$D(2^{1/2}S_0)$	π	2	393	10.8
$D(2^{1/2}S_1)$	π	2	296	5.8
$D(1^{3/2}D_2)$	π	0	142	3.9
$D_s(1^{1/2}S_0)$	K	2	779	15.3
$D_s(1^{1/2}S_1)$	K	2	658	8.7

TABLE XII. (*Continued*).

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(2^{1/2}P_1)$		$m=3.045$ GeV	
$D(1^{1/2}S_1)$	η	0	730	10.9
$D(1^{1/2}S_1)$	π	0	855	85.4
$D(1^{1/2}P_0)$	π	1	582	61.0
$D(1^{3/2}P_1)$	π	1	549	6.6
$D(1^{3/2}P_2)$	π	1	513	19.9
$D(1^{1/2}P_1)$	π	1	488	61.5
$D(2^{1/2}S_1)$	π	0	306	50.6
$D_s(1^{1/2}S_1)$	K	0	667	59.3

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(1^{7/2}F_3)$		$m=3.074$ GeV	
$D(1^{1/2}S_1)$	η	4	754	0.5
$D(1^{1/2}S_1)$	π	4	875	13.1
$D(1^{1/2}P_0)$	π	3	606	11.4
$D(1^{3/2}P_1)$	π	3	573	7.3
$D(1^{3/2}P_2)$	π	3	538	3.8
$D(1^{1/2}P_1)$	π	3	513	5.0
$D(1^{5/2}D_2)$	π	2	251	2.9
$D(1^{3/2}D_1)$	π	2	230	3.3
$D_s(1^{1/2}S_1)$	K	4	693	1.5

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(1^{7/2}F_4)$		$m=3.091$ GeV	
$D(1^{1/2}S_0)$	η	4	872	0.6
$D(1^{1/2}S_0)$	π	4	974	9.8
$D(1^{1/2}S_1)$	π	4	887	7.8
$D(1^{3/2}P_1)$	π	3	587	4.4
$D(1^{3/2}P_2)$	π	3	552	9.9
$D(1^{1/2}P_1)$	π	3	527	9.6
$D(1^{5/2}D_2)$	π	2	268	0.6
$D(1^{5/2}D_3)$	π	2	244	2.9
$D(1^{3/2}D_2)$	π	2	208	2.4
$D_s(1^{1/2}S_0)$	K	4	824	2.0
$D_s(1^{1/2}S_1)$	K	4	707	0.9

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(1^{5/2}F_2)$		$m=3.101$ GeV	
$D(1^{1/2}S_0)$	π	2	981	3.7
$D(1^{1/2}S_1)$	π	2	895	1.8
$D(1^{3/2}P_1)$	η	1	362	3.7
$D(1^{3/2}P_1)$	π	1	595	22.2
$D(1^{3/2}P_1)$	π	3	595	1.4
$D(1^{3/2}P_2)$	π	1	560	2.2
$D(1^{3/2}P_2)$	π	3	560	2.1
$D(1^{1/2}P_1)$	π	3	536	2.6
$D(1^{5/2}D_2)$	π	0	279	56.8
$D(1^{5/2}D_2)$	π	2	279	3.4
$D_s(1^{1/2}S_0)$	K	2	832	3.1
$D_s(1^{1/2}S_1)$	K	2	716	1.2
$D_s(1^{3/2}P_1)$	K	1	250	6.0

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(1^{52}F_3)$		$m=3.123$ GeV	
$D(1^{1/2}S_1)$	π	2	910	4.9
$D(1^{1/2}P_0)$	π	3	645	4.3
$D(1^{3/2}P_1)$	π	3	613	3.2
$D(1^{3/2}P_2)$	η	1	332	3.4
$D(1^{3/2}P_2)$	π	1	579	23.7
$D(1^{3/2}P_2)$	π	3	579	2.6
$D(1^{1/2}P_1)$	π	3	554	1.2
$D(1^{5/2}D_2)$	π	2	300	1.2
$D(1^{5/2}D_3)$	π	0	277	56.7
$D(1^{5/2}D_3)$	π	2	277	3.6
$D_s(1^{1/2}S_1)$	K	2	735	3.3
$D_s(1^{3/2}P_2)$	K	1	203	3.7

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(3^{1/2}S_0)$		$m=3.141$ GeV	
$D(1^{1/2}S_1)$	η	1	811	1.7
$D(1^{1/2}S_1)$	π	1	923	44.5
$D(1^{1/2}P_0)$	π	0	660	2.9
$D(1^{3/2}P_2)$	π	2	594	15.1
$D(2^{1/2}S_1)$	π	1	396	16.3
$D(2^{1/2}P_0)$	π	0	128	6.5
$D_s(1^{1/2}S_1)$	K	1	751	8.7
$D_s(1^{1/2}P_0)$	K	0	384	1.4

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D(3^{1/2}S_1)$		$m=3.226$ GeV	
$D(1^{1/2}S_0)$	η	1	976	3.2
$D(1^{1/2}S_0)$	π	1	1066	50.2
$D(1^{1/2}S_1)$	π	1	984	53.5
$D(1^{3/2}P_1)$	π	2	697	23.1
$D(1^{3/2}P_2)$	π	2	663	17.0
$D(2^{1/2}S_0)$	π	1	560	41.3
$D(2^{1/2}S_1)$	π	1	473	33.5
$D(2^{1/2}P_1)$	π	0	113	5.6
$D_s(1^{1/2}S_0)$	K	1	930	17.6
$D_s(1^{1/2}S_1)$	K	1	822	13.4

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(2^{1/2}S_0)$		$m=2.700$ GeV	
$D(1^{1/2}S_1)$	K	1	424	3.12
$D_s(1^{1/2}S_1)$	η	1	187	0.04

TABLE XII. (*Continued*).

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(2^{1/2}S_1)$		$m=2.806$ GeV	
$D(1^{1/2}S_0)$	K	1	661	21.1
$D(1^{1/2}S_1)$	K	1	539	12.2
$D_s(1^{1/2}S_0)$	η	1	540	6.2
$D_s(1^{1/2}S_1)$	η	1	371	1.5
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{5/2}D_2)$		$m=2.900$ GeV	
$D(1^{1/2}S_1)$	K	3	629	10.6
$D_s(1^{1/2}S_1)$	η	3	486	1.4
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{3/2}D_1)$		$m=2.913$ GeV	
$D(1^{1/2}S_0)$	K	1	752	26.1
$D(1^{1/2}S_1)$	K	1	641	10.7
$D(1^{3/2}P_1)$	K	0	47	39.7
$D_s(1^{1/2}S_0)$	η	1	644	15.2
$D_s(1^{1/2}S_1)$	η	1	501	4.5
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{5/2}D_3)$		$m=2.925$ GeV	
$D(1^{1/2}S_0)$	K	3	762	11.4
$D(1^{1/2}S_1)$	K	3	652	7.3
$D_s(1^{1/2}S_0)$	η	3	656	3.1
$D_s(1^{1/2}S_1)$	η	3	514	1.1
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{3/2}D_2)$		$m=2.953$ GeV	
$D(1^{1/2}S_1)$	K	1	677	35.0
$D_s(1^{1/2}S_1)$	η	1	543	16.4
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(2^{1/2}P_0)$		$m=3.067$ GeV	
$D(1^{1/2}S_0)$	K	0	875	74.1
$D(1^{3/2}P_1)$	K	1	377	32.3
$D(1^{1/2}P_1)$	K	1	270	17.8
$D_s(1^{1/2}S_0)$	η	0	780	49.1
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(2^{3/2}P_1)$		$m=3.114$ GeV	
$D(1^{1/2}S_1)$	K	0	813	(*) 0.7
$D(1^{1/2}S_1)$	K	2	813	36.6
$D(1^{1/2}P_0)$	K	1	482	5.5
$D(1^{3/2}P_1)$	K	1	436	1.6
$D(1^{3/2}P_2)$	K	3	383	0.9
$D(1^{1/2}P_1)$	K	1	343	4.5
$D_s(1^{1/2}S_1)$	η	2	700	11.1
$D_s(1^{1/2}P_0)$	η	1	275	1.6
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(2^{3/2}P_2)$		$m=3.157$ GeV	
$D(1^{1/2}S_0)$	K	2	943	34.9
$D(1^{1/2}S_1)$	K	2	848	28.9
$D(1^{3/2}P_1)$	K	3	485	1.8
$D(1^{1/2}P_1)$	K	1	400	11.0
$D(2^{1/2}S_0)$	K	2	255	1.3
$D_s(1^{1/2}S_0)$	η	2	854	14.8
$D_s(1^{1/2}S_1)$	η	2	738	9.5
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(2^{1/2}P_1)$		$m=3.165$ GeV	
$D(1^{1/2}S_1)$	K	0	854	72.1
$D(1^{1/2}P_0)$	K	1	537	42.6
$D(1^{3/2}P_1)$	K	1	494	12.6
$D(1^{3/2}P_2)$	K	1	446	41.3
$D(1^{1/2}P_1)$	K	1	410	28.7
$D_s(1^{1/2}S_1)$	η	0	745	45.1
$D_s(1^{1/2}P_0)$	η	1	357	8.1
$D_s(1^{3/2}P_1)$	η	1	280	2.7
$D_s(1^{3/2}P_2)$	η	1	184	3.4
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{7/2}F_3)$		$m=3.203$ GeV	
$D(1^{1/2}S_1)$	K	4	884	9.2
$D(1^{1/2}P_0)$	K	3	575	5.5
$D(1^{3/2}P_1)$	K	3	534	3.2
$D(1^{3/2}P_2)$	K	3	489	1.5
$D(1^{1/2}P_1)$	K	3	456	1.6
$D_s(1^{1/2}S_1)$	η	4	778	2.2
$D_s(1^{1/2}P_0)$	η	3	409	0.5
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{7/2}F_4)$		$m=3.220$ GeV	
$D(1^{1/2}S_0)$	K	4	990	7.2
$D(1^{1/2}S_1)$	K	4	897	5.6
$D(1^{3/2}P_1)$	K	3	552	2.0
$D(1^{3/2}P_2)$	K	3	508	4.0
$D(1^{1/2}P_1)$	K	3	475	3.3
$D_s(1^{1/2}S_0)$	η	4	904	2.4
$D_s(1^{1/2}S_1)$	η	4	793	1.4

TABLE XII. (*Continued*).

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{5/2}F_2)$		$m=3.224$ GeV	
$D(1^{1/2}S_0)$	K	2	993	5.4
$D(1^{1/2}S_1)$	K	2	900	3.1
$D(1^{3/2}P_1)$	K	1	556	39.1
$D(1^{3/2}P_1)$	K	3	556	0.9
$D(1^{3/2}P_2)$	K	1	512	4.0
$D(1^{3/2}P_2)$	K	3	512	1.1
$D(1^{1/2}P_1)$	K	3	480	0.9
$D_s(1^{1/2}S_0)$	η	2	907	3.1
$D_s(1^{1/2}S_1)$	η	2	796	1.5
$D_s(1^{3/2}P_1)$	η	1	372	14.0
$D_s(1^{3/2}P_2)$	η	1	302	0.9
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(1^{5/2}F_3)$		$m=3.247$ GeV	
$D(1^{1/2}S_1)$	K	2	918	8.2
$D(1^{1/2}P_0)$	K	3	619	2.2
$D(1^{3/2}P_1)$	K	3	580	2.0
$D(1^{3/2}P_2)$	K	1	536	42.2
$D(1^{3/2}P_2)$	K	3	536	1.5
$D_s(1^{1/2}S_1)$	η	2	815	3.9
$D_s(1^{3/2}P_2)$	η	1	339	12.6
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(3^{1/2}S_0)$		$m=3.259$ GeV	
$D(1^{1/2}S_1)$	K	1	927	19.0
$D(1^{1/2}P_0)$	K	0	630	2.1
$D(1^{3/2}P_2)$	K	2	549	5.1
$D(2^{1/2}S_1)$	K	1	254	1.0
$D_s(1^{1/2}S_1)$	η	1	825	6.8
$D_s(1^{1/2}P_0)$	η	0	478	0.9
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=D_s(3^{1/2}S_1)$		$m=3.345$ GeV	
$D(1^{1/2}S_0)$	K	1	1080	28.0
$D(1^{1/2}S_1)$	K	1	993	26.6
$D(1^{3/2}P_1)$	K	2	675	12.2
$D(1^{3/2}P_2)$	K	2	635	8.1
$D(1^{1/2}P_1)$	K	0	607	1.8
$D(2^{1/2}S_0)$	K	1	507	18.8
$D(2^{1/2}S_1)$	K	1	385	8.4
$D_s(1^{1/2}S_0)$	η	1	1001	13.6
$D_s(1^{1/2}S_1)$	η	1	896	10.6
$D_s(1^{3/2}P_1)$	η	2	523	1.7
$D_s(2^{1/2}S_0)$	η	1	309	1.7
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(2^{1/2}S_0)$		$m=5.886$ GeV	
$B(1^{1/2}S_1)$	π	1	519	21.9
$B(1^{1/2}P_0)$	π	0	114	4.9
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(2^{1/2}S_1)$		$m=5.920$ GeV	
$B(1^{1/2}S_0)$	π	1	591	19.2
$B(1^{1/2}S_1)$	π	1	550	22.9
$B(1^{3/2}P_1)$	π	2	167	0.5
$B(1^{1/2}P_1)$	π	0	109	4.6
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{5/2}D_2)$		$m=5.985$ GeV	
$B(1^{1/2}S_1)$	π	3	611	17.4
$B(1^{3/2}P_1)$	π	2	244	1.8
$B(1^{1/2}P_0)$	π	2	237	1.6
$B(1^{3/2}P_2)$	π	2	228	0.7
$B(1^{1/2}P_1)$	π	2	195	0.5
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{5/2}D_3)$		$m=5.993$ GeV	
$B(1^{1/2}S_0)$	π	3	658	11.1
$B(1^{1/2}S_1)$	π	3	618	10.6
$B(1^{3/2}P_1)$	π	2	252	0.6
$B(1^{3/2}P_2)$	π	2	237	2.2
$B(1^{1/2}P_1)$	π	2	204	1.3
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{3/2}D_1)$		$m=6.025$ GeV	
$B(1^{1/2}S_0)$	η	1	475	2.8
$B(1^{1/2}S_0)$	π	1	687	18.2
$B(1^{1/2}S_1)$	π	1	647	7.5
$B(1^{3/2}P_1)$	π	0	286	81.4
$B(1^{3/2}P_1)$	π	2	286	1.4
$B_s(1^{1/2}S_0)$	K	1	403	7.2
$B_s(1^{1/2}S_1)$	K	1	330	2.0
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{3/2}D_2)$		$m=6.037$ GeV	
$B(1^{1/2}S_1)$	η	1	431	3.3
$B(1^{1/2}S_1)$	π	1	658	23.9
$B(1^{3/2}P_1)$	π	2	299	1.3
$B(1^{3/2}P_2)$	π	0	284	81.1
$B(1^{3/2}P_2)$	π	2	284	2.0
$B_s(1^{1/2}S_1)$	K	1	350	7.1

TABLE XII. (*Continued*).

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(2^{1/2}P_0)$	$m=6.163$ GeV		
$B(1^{1/2}S_0)$	η	0	643	9.9
$B(1^{1/2}S_0)$	π	0	810	97.4
$B(1^{3/2}P_1)$	π	1	425	21.1
$B(1^{1/2}P_1)$	π	1	383	52.9
$B(2^{1/2}S_0)$	π	0	233	39.6
$B_s(1^{1/2}S_0)$	K	0	576	51.0

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(2^{3/2}P_1)$	$m=6.175$ GeV		
$B(1^{1/2}S_1)$	η	2	606	1.5
$B(1^{1/2}S_1)$	π	2	782	59.6
$B(1^{3/2}P_1)$	π	1	437	1.6
$B(1^{1/2}P_0)$	π	1	431	2.4
$B(1^{3/2}P_2)$	π	3	423	2.6
$B(1^{1/2}P_1)$	π	1	395	3.4
$B(2^{1/2}S_1)$	π	2	209	2.3
$B(1^{3/2}D_1)$	π	0	55	1.4
$B_s(1^{1/2}S_1)$	K	2	534	4.6

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(2^{3/2}P_2)$	$m=6.188$ GeV		
$B(1^{1/2}S_0)$	η	2	671	1.2
$B(1^{1/2}S_0)$	π	2	832	36.0
$B(1^{1/2}S_1)$	η	2	621	1.1
$B(1^{1/2}S_1)$	π	2	793	39.7
$B(1^{3/2}P_1)$	π	3	449	2.2
$B(1^{3/2}P_2)$	π	3	436	1.2
$B(1^{1/2}P_1)$	π	1	408	5.7
$B(2^{1/2}S_0)$	π	2	261	2.4
$B(2^{1/2}S_1)$	π	2	224	1.9
$B(1^{3/2}D_2)$	π	0	57	1.5
$B_s(1^{1/2}S_0)$	K	2	605	4.2
$B_s(1^{1/2}S_1)$	K	2	550	3.4

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(2^{1/2}P_1)$	$m=6.194$ GeV		
$B(1^{1/2}S_1)$	η	0	629	9.2
$B(1^{1/2}S_1)$	π	0	799	93.2
$B(1^{3/2}P_1)$	π	1	455	6.1
$B(1^{1/2}P_0)$	π	1	449	30.6
$B(1^{3/2}P_2)$	π	1	442	22.6
$B(1^{1/2}P_1)$	π	1	414	39.5
$B(2^{1/2}S_1)$	π	0	231	38.7
$B_s(1^{1/2}S_1)$	K	0	557	46.3

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{7/2}F_3)$	$m=6.220$ GeV		
$B(1^{1/2}S_1)$	η	4	659	0.3
$B(1^{1/2}S_1)$	π	4	822	11.7
$B(1^{3/2}P_1)$	π	3	481	3.5
$B(1^{1/2}P_0)$	π	3	475	3.8
$B(1^{3/2}P_2)$	π	3	468	1.9
$B(1^{1/2}P_1)$	π	3	440	2.1
$B(1^{5/2}D_2)$	π	2	186	0.8
$B(1^{3/2}D_1)$	π	2	135	0.3
$B_s(1^{1/2}S_1)$	K	4	588	0.5

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{7/2}F_4)$	$m=6.226$ GeV		
$B(1^{1/2}S_0)$	π	4	865	7.0
$B(1^{1/2}S_1)$	π	4	827	6.7
$B(1^{3/2}P_1)$	π	3	486	1.5
$B(1^{3/2}P_2)$	π	3	473	4.7
$B(1^{1/2}P_1)$	π	3	445	4.4
$B(1^{5/2}D_3)$	π	2	183	0.8
$B_s(1^{1/2}S_0)$	K	4	647	0.5
$B_s(1^{1/2}S_1)$	K	4	594	0.3

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{5/2}F_2)$	$m=6.264$ GeV		
$B(1^{1/2}S_0)$	π	2	898	3.7
$B(1^{1/2}S_1)$	π	2	860	2.0
$B(1^{3/2}P_1)$	π	1	522	18.9
$B(1^{3/2}P_2)$	π	1	509	2.0
$B(1^{3/2}P_2)$	π	3	509	1.3
$B(1^{1/2}P_1)$	π	3	482	1.3
$B(1^{5/2}D_2)$	π	0	236	51.2
$B(1^{5/2}D_2)$	π	2	236	1.6
$B_s(1^{1/2}S_0)$	K	2	688	1.9
$B_s(1^{1/2}S_1)$	K	2	637	0.9

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(1^{5/2}F_3)$	$m=6.271$ GeV		
$B(1^{1/2}S_1)$	η	2	714	1.0
$B(1^{1/2}S_1)$	π	2	866	5.2
$B(1^{3/2}P_1)$	π	3	529	1.2
$B(1^{1/2}P_0)$	π	3	523	1.0
$B(1^{3/2}P_2)$	π	1	516	20.4
$B(1^{3/2}P_2)$	π	3	516	1.5
$B(1^{5/2}D_3)$	π	0	235	51.0
$B(1^{5/2}D_3)$	π	2	235	1.7
$B_s(1^{1/2}S_1)$	K	2	644	2.3

TABLE XII. (*Continued*).

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(3^{1/2}S_0)$	$m=6.320$ GeV		
$B(1^{1/2}S_1)$	η	1	766	1.1
$B(1^{1/2}S_1)$	π	1	908	49.5
$B(1^{1/2}P_0)$	π	0	569	1.8
$B(1^{3/2}P_2)$	π	2	562	11.0
$B(2^{1/2}S_1)$	π	1	363	9.5
$B(2^{1/2}P_0)$	π	0	72	3.5
$B_s(1^{1/2}S_1)$	K	1	697	5.1

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B(3^{1/2}S_1)$	$m=6.347$ GeV		
$B(1^{1/2}S_0)$	π	1	970	31.3
$B(1^{1/2}S_1)$	π	1	932	43.0
$B(1^{3/2}P_1)$	π	2	600	9.4
$B(1^{3/2}P_2)$	π	2	587	7.9
$B(1^{1/2}P_1)$	π	0	560	1.7
$B(2^{1/2}S_0)$	π	1	423	9.3
$B(2^{1/2}S_1)$	π	1	390	10.8
$B(2^{1/2}P_1)$	π	0	64	3.0
$B_s(1^{1/2}S_0)$	K	1	775	5.0
$B_s(1^{1/2}S_1)$	K	1	726	5.2

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(2^{1/2}S_0)$	$m=5.985$ GeV		
$B(1^{1/2}S_1)$	K	1	416	3.15

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(2^{1/2}S_1)$	$m=6.019$ GeV		
$B(1^{1/2}S_0)$	K	1	517	5.6
$B(1^{1/2}S_1)$	K	1	462	4.9
$B_s(1^{1/2}S_0)$	η	1	325	0.4

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{5/2}D_2)$	$m=6.095$ GeV		
$B(1^{1/2}S_1)$	K	3	555	6.62
$B_s(1^{1/2}S_1)$	η	3	370	0.30

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{5/2}D_3)$	$m=6.103$ GeV		
$B(1^{1/2}S_0)$	K	3	614	5.03
$B(1^{1/2}S_1)$	K	3	564	4.17
$B_s(1^{1/2}S_0)$	η	3	453	0.46
$B_s(1^{1/2}S_1)$	η	3	383	0.21

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{3/2}D_1)$	$m=6.127$ GeV		
$B(1^{1/2}S_0)$	K	1	641	26.9
$B(1^{1/2}S_1)$	K	1	592	11.5
$B_s(1^{1/2}S_0)$	η	1	486	10.1
$B_s(1^{1/2}S_1)$	η	1	420	3.4

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{3/2}D_2)$	$m=6.140$ GeV		
$B(1^{1/2}S_1)$	K	1	607	36.2
$B_s(1^{1l/2}S_1)$	η	1	438	11.5

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(2^{1/2}P_0)$	$m=6.264$ GeV		
$B(1^{1/2}S_0)$	K	0	785	77.7
$B(1^{3/2}P_1)$	K	1	262	20.6
$B(1^{1/2}P_1)$	K	1	164	3.8
$B_s(1^{1/2}S_0)$	η	0	652	40.5

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(2^{3/2}P_1)$	$m=6.278$ GeV		
$B(1^{1/2}S_1)$	K	0	755	(*) 0.7
$B(1^{1/2}S_1)$	K	2	755	28.7
$B(1^{1/2}P_0)$	K	1	278	3.3
$B(1^{1/2}P_1)$	K	1	201	1.4
$B_s(1^{1/2}S_1)$	η	2	614	5.8

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(2^{3/2}P_2)$	$m=6.292$ GeV		
$B(1^{1/2}S_0)$	K	2	813	19.2
$B(1^{1/2}S_1)$	K	2	769	19.8
$B(1^{1/2}P_1)$	K	1	232	4.2
$B_s(1^{1/2}S_0)$	η	2	683	4.8
$B_s(1^{1/2}S_1)$	η	2	630	4.2

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(2^{1/2}P_1)$	$m=6.296$ GeV		
$B(1^{1/2}S_1)$	K	0	773	73.8
$B(1^{3/2}P_1)$	K	1	319	7.0
$B(1^{1/2}P_0)$	K	1	309	9.3
$B(1^{3/2}P_2)$	K	1	296	24.5
$B(1^{1/2}P_1)$	K	1	241	6.0
$B_s(1^{1/2}S_1)$	η	0	634	37.1

TABLE XII. (*Continued*).

$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{7/2}F_3)$		$m=6.332$ GeV	
$B(1^{1/2}S_1)$	K	4	808	7.07
$B(1^{3/2}P_1)$	K	3	375	0.51
$B(1^{1/2}P_0)$	K	3	366	0.50
$B(1^{3/2}P_2)$	K	3	354	0.22
$B(1^{1/2}P_1)$	K	3	307	0.14
$B_s(1^{1/2}S_1)$	η	4	674	1.04
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{7/2}F_4)$		$m=6.337$ GeV	
$B(1^{1/2}S_0)$	K	4	857	4.5
$B(1^{1/2}S_1)$	K	4	814	4.1
$B(1^{3/2}P_1)$	K	3	383	0.2
$B(1^{3/2}P_2)$	K	3	363	0.6
$B(1^{1/2}P_1)$	K	3	317	0.3
$B_s(1^{1/2}S_0)$	η	4	732	0.8
$B_s(1^{1/2}S_1)$	η	4	681	0.6
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{5/2}F_2)$		$m=6.369$ GeV	
$B(1^{1/2}S_0)$	K	2	888	6.0
$B(1^{1/2}S_1)$	K	2	845	3.5
$B(1^{3/2}P_1)$	K	1	428	29.6
$B(1^{3/2}P_2)$	K	1	409	3.0
$B_s(1^{1/2}S_0)$	η	2	766	2.4
$B_s(1^{1/2}S_1)$	η	2	715	1.3
$B_s(1^{3/2}P_1)$	η	1	129	0.9
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(1^{5/2}F_3)$		$m=6.376$ GeV	
$B(1^{1/2}S_1)$	K	2	853	9.0
$B(1^{3/2}P_2)$	K	1	420	31.6
$B_s(1^{1/2}S_1)$	η	2	723	3.3
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(3^{1/2}S_0)$		$m=6.421$ GeV	
$B(1^{1/2}S_1)$	K	1	896	16.5
$B(1^{1/2}P_0)$	K	0	489	0.3
$B(1^{3/2}P_2)$	K	2	479	1.7
$B_s(1^{1/2}S_1)$	η	1	770	3.9
$H(n^j l_J)$	x	l_x	p_x	$\Gamma_x/(g_A^8)^2$
	$H'=B_s(3^{1/2}S_1)$		$m=6.449$ GeV	
$B(1^{1/2}S_0)$	K	1	963	13.2
$B(1^{1/2}S_1)$	K	1	922	16.0
$B(1^{3/2}P_1)$	K	2	531	2.2
$B(1^{3/2}P_2)$	K	2	514	1.7
$B_s(1^{1/2}S_0)$	η	1	848	3.9
$B_s(1^{1/2}S_1)$	η	1	800	4.1

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