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B Lifetimes from Lattice Simulations*

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We evaluate the matrix elements of the four-quark operators which contribute to the lifetimes of B-mesons and the Λ_b-baryon. We find that the spectator effects are indeed larger than naive expectations based purely on power counting even if they do not appear to be sufficiently large to fully account for the discrepancy between the $O(1/m_b^2)$ theoretical prediction and experimental measurement of the ratio of lifetimes $\tau(\Lambda_b)/\tau(B)$.

1. INTRODUCTION

Inclusive decays of heavy hadrons can be studied in the framework of the heavy quark expansion, in which lifetimes are computed as series in inverse powers of the mass of the b-quark [3]. For an arbitrary hadron H

$$\tau^{-1}(H) = \frac{G_F^2 m_b^5}{192\pi^3} \frac{|V_{cb}|^2}{2m_H} \sum_{i\geq 0} c_i m_b^{-i} \tag{1}$$

where

- c_0 corresponds to the decay of a free-quark and is universal.
- c_1 is zero because the operators of dimension four can be eliminated using the equations of motion.
- c_2 can be estimated and is found to be small.
- c_3 contains a contribution proportional to

$$\langle H|\ \bar{b}\Gamma q\ \bar{q}\widetilde{\Gamma} b\ |H\rangle \tag{2}$$

The term c_2 contains an ambiguity due to the fact that when it is factorized in a perturbative part times a non perturbative one, the latter contains a divergence which is cancelled by the former only at an infinite order in perturbation theory. In order to avoid complications due to renormalon ambiguities we consider ratios of lifetimes. The differences in these ratios due to operators of dimension 5 can be extimated and are found to be small.

The term of eq. (2) originates from integrating out the internal excitations of the box diagram in which two fermionic lines exchange two W. This is the first term in the expansion to which the interaction between the heavy quark and the constituent light (anti)quark contribute. Although this is an $O(m_b^{-3})$ correction, it may be significant since it contains a phase-space enhancement.

The aim of our work is to compute (making use of lattice simulations) these spectator contributions to c_3 for B_q-mesons and the Λ_b-baryon. The experimental values for the ratios of lifetimes of these particles are

$$\frac{\tau(B^-)}{\tau(B^0)} = 1.06 \pm 0.04 \tag{3}$$

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.78 \pm 0.04\ . \tag{4}$$

The discrepancy between the experimental value in eq. (4) and the theoretical prediction of $\tau(\Lambda_b)/\tau(B^0) = 0.98$ (based on the Operator Product Expansion in eq. (1) including terms in the sum up to those of $O(m_b^{-2})$) is a major puzzle. It is therefore particularly important to compute the $O(m_b^{-3})$ spectator contributions to this ratio.

The ratios in eqs. (3) and (4) can be expressed in terms of 6 matrix elements:

$$\frac{\tau(B^-)}{\tau(B^0)} = a_0 + a_1\varepsilon_1 + a_2\varepsilon_2 + a_3 B_1 + a_3 B_2 \tag{5}$$

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = b_0 + b_1\varepsilon_1 + b_2\varepsilon_2 + b_3 L_1 + b_4 L_2 \tag{6}$$

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where [2]

$$B_1 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\,\bar{b}\gamma^\mu L q\,\bar{q}\gamma_\mu L b\,|B\rangle}{2m_B} \tag{7}$$

$$B_2 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\,\bar{b}L q\,\bar{q}R b\,|B\rangle}{2m_B} \tag{8}$$

$$\varepsilon_1 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\,\bar{b}\gamma^\mu L t^a q\,\bar{q}\gamma_\mu L t^a b\,|B\rangle}{2m_B} \tag{9}$$

$$\varepsilon_2 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\,\bar{b}L t^a q\,\bar{q}R t^a b\,|B\rangle}{2m_B} \tag{10}$$

$$L_1 \equiv \frac{8}{f_B^2 m_B} \frac{\langle \Lambda|\,\bar{b}\gamma^\mu L q\,\bar{q}\gamma_\mu L b\,|\Lambda\rangle}{2m_\Lambda} \tag{11}$$

$$L_2 \equiv \frac{8}{f_B^2 m_B} \frac{\langle \Lambda|\,\bar{b}\gamma^\mu L t^a q\,\bar{q}\gamma_\mu L t^a b\,|\Lambda\rangle}{2m_\Lambda} \tag{12}$$

and the coefficients a_i and b_i are given by

	value		value
a_0	+1.00	b_0	+0.98
a_1	−0.697	b_1	−0.173
a_2	+0.195	b_2	+0.195
a_3	+0.020	b_3	+0.030
a_4	+0.004	b_4	−0.252

The values in the table correspond to operators renormalized in the continuum $\overline{\text{MS}}$ renormalization scheme at the scale $\mu = m_B$.

2. GENERAL REMARKS

Matrix elements such as those of eqs. (7-12) encode the contribution of soft QCD effects which cannot be evaluated in perturbation theory. These effects can be computed by performing a numerical evaluation of the path integral that defines the matrix elements. The standard procedure consists in approximating a portion of continuum space-time with a discrete four dimensional lattice endowed with an Euclidean metric, and formulating QCD on this lattice. The theory, without any model dependent assumption, presents an exact gauge invariance. The discrepancy between the discretized correlation functions and the contunuum ones is a function of the lattice spacing (a) that can (in principle) be systematically reduced.

The lattice provides a natural momentum cutoff because modes with frequency higher than the inverse lattice spacing cannot propagate. The limit $a \to 0$ can be taken and it becomes evident how the lattice formulation of QCD is just a way of regularizing the theory. Moreover for any finite value of a, lattice QCD can be thought of as an effective theory of QCD: the effects of heavy modes are encoded in its parameters (which can be evaluated by direct comparison with phenomenology) and in the matching coefficients (which can be evaluted in perturbation theory) that multiply the lattice matrix elements (which are computed numerically).

To relate the matrix elements of eqs. (7-12) to the lattice ones, it is necessary to evolve them down to an energy scale that can be simulated, a^{-1} (using the renormalization group equation), and then match them with the matrix elements regularized on the lattice (and renormalized at the lattice scale, a). In our simulations we adopt a Sheikoleslami-Wolhert (SW) lattice action which improves the convergence of the correlation functions to those of continuum QCD form order $O(a)$ to order $O(\alpha_s a)$. We compute the corresponding matching coefficients at one loop. Our ignorance of higher order coefficients introduces a systematic error that we include in our analysis.

Moreover because the lattice describes only a finite portion of space-time, massless modes cannot propagate even. For this reason the light quarks (u and d) are simulated with higher masses,

$$m_{\text{light}} \simeq (2a)^{-1}(\kappa^{-1} - \kappa_{crit}^{-1}) \tag{13}$$

for several values fo the parameter κ. Then the results are extrapolated to the chiral limit ($\kappa \to \kappa_{crit}$). This procedure introduces an error that, for the matrix elements of eqs. (7-10), has been evaluated. For the matrix elements of eqs. (11-12) this extrapolation has not been possible because only two values for the light quark masses are available at the moment.

The most important systematic error intro-

[2] In terms of the parameters $\widetilde{B}$ and r introduced in ref. [4]

$$r = -6L_1$$
$$\widetilde{B} = -2L_2/L_1 - 1/3$$

duced in lattice simulations is the quenched approximation, i.e. quark loops are neglected. This is due to limitations in present computing power and in principle the effects of loops could be evalueted in future simulations. The size of this systematic error is not known and is not quoted in our results.

We have adopted the following standard technique to extract matrix elements of the form $\langle H|O_i|H\rangle$ from the lattice. We construct a lattice operator $J^\dagger$ which has the same quantum numbers as the hadron H, where H is the lightest state with a non vanishing superposition with $J^\dagger$. We evaluate on the lattice the zero momentum Fourier transform of the 2-point correlation function:

$$C_2(t) = \int \mathrm{d}^3x \langle 0|J(\mathbf{x},t)J^\dagger(\mathbf{x},0))|0\rangle \tag{14}$$

and of the 3-point one:

$$C_{3i}(t) = \int \mathrm{d}^3x \langle 0|J(\mathbf{x},t)O_i(\mathbf{x},0)J^\dagger(\mathbf{x},-t))|0\rangle \tag{15}$$

In the Euclidean space, for large t, these correlation functions are dominated by the lightest particle created by $J^\dagger$, i.e. H, therefore

$$\lim_{t\to\infty} Z_{ij} Z_J^2 \frac{C_{3j}(t)}{[C_2(t)]^2} = \frac{\langle H|O_i|H\rangle}{2m_H} \tag{16}$$

enables us to extract the required matrix element.

The factor Z_J measures the superposition between H and $J^\dagger$ and it can be extracted by fitting the large t behaviour of $C_2(t)$ with a single exponential

$$C_2(t) \simeq Z_J e^{mt} \tag{17}$$

The coefficients $Z_{ij} = \delta_{ij} + O(\alpha_s)$ are the matching coefficients that we have evaluated perturbatively.

In general there are many possible choices for the smearing of the interpolation operator J. In the case of the operators of eqs. (7-10), we checked that the matrix elements do not depend on the smearing procedure.

3. B DECAY

The matrix elements $B_1, B_2, \varepsilon_1, \varepsilon_2$ are computed on a $24^3\times 48$ lattice at $\beta = 6.2$ (corresponding to a lattice spacing $a^{-1} = 2.9(1)$ GeV) using the tree-level improved SW action for three values of $\kappa = 0.14144, 0.14226, 0.14262$ and are then extrapolated to the chiral limit ($\kappa_{crit} = 0.14315$) [1]. We find

$$B_1 = +1.06 \pm 0.08 \tag{18}$$

$$B_2 = +1.01 \pm 0.06 \tag{19}$$

$$\varepsilon_1 = -0.01 \pm 0.03 \tag{20}$$

$$\varepsilon_2 = -0.02 \pm 0.02 \tag{21}$$

which implies that

$$\frac{\tau(B^-)}{\tau(B^0)} = 1.03 \pm 0.02 \pm 0.03 \tag{22}$$

in agreement with the experimental value, eq. (3). In eq. (22) the first error is purely statistical while the second one encodes our evaluation for systematical uncertainties (excluding quenching effects).

4. Λ DECAY

The computation the baryonic matrix elements L_1 and L_2 is a little more difficult because of the presence of two constituent light quarks in Λ_b. We have performed an exploratory study in which the light quark propagators are computed using a stochastic method [5] based on the relation

$$M_{ij}^{-1} = \int [\mathrm{d}\phi](M_{jk}\phi_k)^*\phi_i e^{-\phi_l^*(M^+M)_{lm}\phi_m} \tag{23}$$

The matrix elements are computed on a $12^3 \times 24$ lattice at $\beta = 5.7$ (corresponding to a lattice spacing $a^{-1} = 1.10(1)$ GeV) for two values of κ. We therefore do not attempt an extrapolation to the chiral limit ($\kappa_{crit} = 0.14351$) but present our results seperately for each value of κ. We find:

$$L_1 = \begin{cases} -0.30 \pm 0.03 & (\kappa = 0.13843) \\ -0.22 \pm 0.03 & (\kappa = 0.14077) \end{cases} \tag{24}$$

$$L_2 = \begin{cases} +0.23 \pm 0.02 & (\kappa = 0.13843) \\ +0.17 \pm 0.02 & (\kappa = 0.14077) \end{cases} , \tag{25}$$

which implies that

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = \begin{cases} 0.91 \pm 0.01 & (\kappa = 0.13843) \\ 0.93 \pm 0.01 & (\kappa = 0.14077) \end{cases} . \tag{26}$$

We stress again that these errors do not include systematical errors due to the chiral extrapolation and to quenching. Our results indicates that spectator effects give rise to a significant difference in the lifetimes of the Λ_b and B. We remark that our computation of the L_1 and L_2 matrix elements must be considered exploratory because of the large lattice spacing, the size of the lattice and the lack of a chiral extrapolation. We believe that it is important to repeat the calculation on a larger lattice with better statistics.

5. NOTES ON FACTORIZATION

While performing our simulation we have found that the matrix elements of the 4-quark operators (18-21) satisfy the vacuum saturation hypothesis (also known as factorization) remarkably well. In fact within statistical errors

$$\langle B_q|\bar{b}\Gamma q\,\bar{q}\tilde{\Gamma}b|B_q\rangle \simeq \langle B_q|\bar{b}\Gamma q|0\rangle\,\langle 0|\bar{q}\tilde{\Gamma}b|B_q\rangle \qquad (27)$$

is verified for any couple, Γ and $\tilde{\Gamma}$, of color $\otimes$ spin matrices. From the theoretical point of view eq. (27) is true at tree level, but no argument is known to prove that it holds at higher orders in perturbation theory or non-perturbatively. In particular factorization cannot hold at every scale since the renormalization group behaviour is different on both sides of eq. (27).

An analogous relation has been observed for the B-$\bar{B}$ system

$$\langle B_q|\bar{b}\Gamma q\,\bar{b}\tilde{\Gamma}q'|\bar{B}_{q'}\rangle \simeq \langle B_q|\bar{b}\Gamma q|0\rangle\,\langle 0|\bar{b}\tilde{\Gamma}q'|\bar{B}_{q'}\rangle \qquad (28)$$

This matrix element contributes as one of the two possible contractions to the matrix element which dominates the mixing amplitude:

$$\begin{aligned}\langle B_q|\bar{b}\Gamma q\,\bar{b}\tilde{\Gamma}q|\bar{B}_q\rangle &= \langle B_q|\bar{b}\Gamma q\,\bar{b}\tilde{\Gamma}q'|\bar{B}_{q'}\rangle \\ &+ \langle B_q|\bar{b}\Gamma q'\,\bar{b}\tilde{\Gamma}q|\bar{B}_{q'}\rangle \qquad (29)\end{aligned}$$

Note that since the second contraction can be reduced to the first by use of the Fierz identities, the assumption of factorization completely determines the total mixing amplitue, eq. (29). Results for $B-\bar{B}$ mixing consistent with factorization have been obtained independently by many groups among the lattice community, and there is general agreement on them.

We believe that the observed phenomenon of factorization constitutes *per se* an interesting result that has to be explained to fully understand the physics of B decays and B-$\bar{B}$ mixing.

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