# An exploratory lattice study of spectator effects in inclusive decays of the $\Lambda_{b}$ baryon 

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#### Abstract

A possible explanation of the apparent discrepancy between the theoretical prediction and experimental measurement of the ratio of lifetimes $\tau\left(\Lambda_{b}\right) / \tau\left(B_{d}\right)$ is that "spectator effects", which appear at $O\left(1 / m_{b}^{3}\right)$ in the heavy quark expansion, contribute significantly. We investigate this possibility by computing the corresponding operator matrix elements in a lattice simulation. We find that spectator effects are indeed significant, but do not appear to be sufficiently large to account for the full discrepancy. We stress, however, that this is an exploratory study, and it is important to check our conclusions on a larger lattice and using a larger sample of gluon configurations. © 1999 Published by Elsevier Science B.V. All rights reserved.


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## 1. Introduction

At leading order in the heavy-quark expansion the decay rate of the heavy quark is independent of its parent hadron. In this letter we present the results of an exploratory study, in which we attempt to gain some understanding of why the experimental result for the ratio of lifetimes [1]
$\frac{\tau\left(\Lambda_{b}\right)}{\tau\left(B_{d}\right)}=0.78 \pm 0.04$
differs so markedly from 1 . In the heavy-quark expansion (see Refs. [2-7] and references therein) this ratio is equal to 1 at leading order, has no corrections of $O\left(1 / m_{b}\right)$ (where $m_{b}$ is the mass of
the $b$-quark) $[4,8]$ and the $O\left(1 / m_{b}^{2}\right)$ corrections can be evaluated to give [9],

$$
\begin{equation*}
\frac{\tau\left(\Lambda_{b}\right)}{\tau\left(B_{d}\right)}=0.98+O\left(1 / m_{b}^{3}\right) . \tag{2}
\end{equation*}
$$

Here we present a lattice computation of the contributions to the $O\left(1 / m_{b}^{3}\right)$ term on the right-hand side of Eq. (2) which come from 'spectator effects', i.e. from decays in which two quark or antiquark constituents of the beauty hadron participate in the weak decay. These effects may be larger than estimates based purely on power counting would indicate as a result of the enhancement of the phase space for $2 \rightarrow 2$ body reactions relative to $1 \rightarrow 3$ body decays.

A comparison of our results with estimates obtained using other techniques is presented at the end of Section 3.

Denoting the $O\left(1 / m_{b}^{3}\right)$ contribution from spectator effects by $\Delta$ and following the analysis of Ref. [9] we find that ${ }^{1}$
$\Delta=-0.173 \varepsilon_{1}+0.195 \varepsilon_{2}+0.030 L_{1}-0.252 L_{2}$,
where

$$
\begin{align*}
\varepsilon_{1}\left(m_{b}\right) & =\frac{8}{f_{B}^{2} m_{B}} \frac{\left\langle B_{q}\right| \bar{b} \gamma^{\mu} L t^{a} q \bar{q} \gamma^{\mu} L t^{a} b\left|B_{q}\right\rangle}{2 m_{B}} \\
& =-0.01 \pm 0.03 \tag{6}
\end{align*}
$$

$$
\varepsilon_{2}\left(m_{b}\right)=\frac{8}{f_{B}^{2} m_{B}} \frac{\left\langle B_{q}\right| \bar{b} L t^{a} q \bar{q} R t^{a} b\left|B_{q}\right\rangle}{2 m_{B}}
$$

$$
=-0.02 \pm 0.02
$$

have been computed, using lattice simulations, in Ref. [10]. The two new variables introduced here are
$L_{1}\left(m_{b}\right)=\frac{8}{f_{B}^{2} m_{B}} \frac{\left\langle\Lambda_{b}\right| \bar{b} \gamma^{\mu} L q \bar{q} \gamma^{\mu} L b\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}}$,
$L_{2}\left(m_{b}\right)=\frac{8}{f_{B}^{2} m_{B}} \frac{\left\langle\Lambda_{b}\right| \bar{b} \gamma^{\mu} L t^{a} q \bar{q} \gamma^{\mu} L t^{a} b\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}}$.
Heavy-quark symmetry implies that there are only two matrix elements which need to be considered for the $\Lambda_{b}$, in contrast to the four for $B$-mesons [9] ${ }^{2}$.

We stress that this is an exploratory study. It is the first calculation of the matrix elements $L_{1,2}$ and provides a preliminary indication of whether spectator effects can reconcile the experimental measurements and theoretical predictions for the ratios of lifetimes in Eqs. (1) and (2). We perform the calculations with a static $b$-quark and two (rather large)

[^0]values of the mass of the light-quark and do not attempt to extrapolate the results to the chiral limit. Our results indicate that spectator effects are not negligible, although they do not appear to be sufficiently large to account fully for the discrepancy. Specifically we find:

$L_{1}\left(m_{b}\right)= \begin{cases}-0.31(3) & \text { for } a m_{\pi}=0.52(3) \\ -0.22(4) & \text { for } a m_{\pi}=0.74(4),\end{cases}$
$L_{2}\left(m_{b}\right)= \begin{cases}0.23(2) & \text { for } a m_{\pi}=0.52(3) \\ 0.17(2) & \text { for } a m_{\pi}=0.74(4),\end{cases}$
with $a^{-1} \simeq 1.1 \mathrm{GeV}$. The corresponding results for the ratio of lifetimes are

$$
\frac{\tau\left(\Lambda_{b}\right)}{\tau\left(B_{d}\right)}= \begin{cases}0.91(1) & \text { for } a m_{\pi}=0.52(3)  \tag{12}\\ 0.93(1) & \text { for } a m_{\pi}=0.74(4)\end{cases}
$$

## 2. Perturbative matching

In this section we briefly discuss the matching factors which are required to obtain the matrix elements of the continuum four-quark operators renormalised at a scale $\mu$ from those of the bare lattice operators computed in lattice simulations at a cut-off $a^{-1}$. The details of the calculation are presented in Ref. [10]. Here we simply summarise the main points required for the evaluation of the matrix elements $L_{1}$ and $L_{2}$ in the $\overline{\mathrm{MS}}$ scheme.

We start by using the renormalisation group to relate the matrix elements $L_{1}$ and $L_{2}$, defined in the $\overline{\mathrm{MS}}$ scheme at two different renormalisation scales, $\mu=m_{b}$ and $\mu=a^{-1}$. Since the Wilson coefficient functions in the OPE expansion (5) have been evaluated at tree level only [9], we keep just the leading logarithms in the evolution equations so that [11,12]

$$
\begin{align*}
\binom{L_{1}\left(m_{b}\right)}{L_{2}\left(m_{b}\right)}= & \left(\begin{array}{cc}
1+\frac{2 C_{F} \delta}{N_{c}} & -\frac{2 \delta}{N_{c}} \\
-\frac{C_{F} \delta}{N_{c}^{2}} & 1+\frac{\delta}{N_{c}^{2}}
\end{array}\right) \\
& \times\binom{ L_{1}\left(a^{-1}\right)}{L_{2}\left(a^{-1}\right)}, \tag{13}
\end{align*}
$$

where
$\delta=\left(\frac{\alpha_{s}^{\overline{\mathrm{MS}}}\left(a^{-1}\right)}{\alpha_{s}^{\mathrm{MS}}\left(m_{b}\right)}\right)^{9 / 2 \beta_{0}}-1=0.40 \pm 0.04$.
In estimating $\delta$ we have used $\Lambda_{\mathrm{QCD}}=250 \mathrm{MeV}$, $a^{-1}=1.10 \mathrm{GeV}, m_{b}=4.5 \mathrm{GeV}$ and $\beta_{0}=9$. The error in $\delta$ is evaluated includes a $20 \%$ uncertainty for $\Lambda_{\mathrm{QCD}}$.

In the second step of the matching we relate the matrix elements renormalised in the continuum to those regularized on lattice, both at the same scale, $a^{-1}$. Although this involves corrections of $O\left(\alpha_{s}\right)$, which are, in principle, beyond the precision which we require, we nevertheless include them because the perturbative coefficients in lattice perturbation theory are generally large. The computation for the most general four quark operator involving one heavy quark appears in the appendix of Ref. [10]. For $L_{1}$ and $L_{2}$ the relevant relations are:

$$
\begin{align*}
& L_{1}\left(a^{-1}\right) \\
& \quad=\frac{8}{f_{B}^{2} m_{B}}\left[h_{11} M_{1}+h_{12} M_{2}+h_{13} M_{3}+h_{14} M_{4}\right],  \tag{15}\\
& L_{2}\left(a^{-1}\right) \\
& \quad=\frac{8}{f_{B}^{2} m_{B}}\left[h_{21} M_{1}+h_{22} M_{2}+h_{23} M_{3}+h_{24} M_{4}\right], \tag{16}
\end{align*}
$$

where

$$
\begin{align*}
M_{1}= & \frac{\left\langle\Lambda_{b}\right|\left(\bar{b} \gamma^{\mu} L q\right)\left(\bar{q} \gamma^{\mu} L b\right)\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}},  \tag{17}\\
M_{2}= & \frac{\left\langle\Lambda_{b}\right|\left(\bar{b} \gamma^{\mu} L \gamma^{0} q\right)\left(\bar{q} \gamma^{\mu} L b\right)\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}} \\
& +\frac{\left\langle\Lambda_{b}\right|\left(\bar{b} \gamma^{\mu} L q\right)\left(\bar{q} \gamma^{0} \gamma^{\mu} L b\right)\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}},  \tag{18}\\
M_{3}= & \frac{\left\langle\Lambda_{b}\right|\left(\bar{b} \gamma^{\mu} L t^{a} q\right)\left(\bar{q} \gamma^{\mu} L t^{a} b\right)\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}}  \tag{19}\\
M_{4}= & \frac{\left\langle\Lambda_{b}\right|\left(\bar{b} \gamma^{\mu} L \gamma^{0} t^{a} q\right)\left(\bar{q} \gamma^{\mu} L t^{a} b\right)\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}} \\
& +\frac{\left\langle\Lambda_{b}\right|\left(\bar{b} \gamma^{\mu} L t^{a} q\right)\left(\bar{q} \gamma^{0} \gamma^{\mu} L t^{a} b\right)\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda_{b}}} \tag{20}
\end{align*}
$$

are the matrix elements of the bare lattice operators regularised at $a^{-1}$. The coefficients $h_{i j}$ are listed in Table 1 where for the lattice coupling constant we have used a boosted coupling equal to

$$
\begin{equation*}
\frac{\alpha_{s}^{\text {latt }}\left(a^{-1}\right)}{4 \pi}=\frac{6\left(8 \kappa_{\text {crit }}\right)^{4}}{(4 \pi)^{2} \beta} \simeq 0.01216 \tag{21}
\end{equation*}
$$

Readers who prefer to use other choices of the lattice coupling constant can combine the coefficients in Table 1 with their choice of coupling.

A consequence of the Heavy Quark Effective Theory, and the fact that the light quarks in the $\Lambda_{b}$ are in a spin zero combination, is that the number of lattice operators whose matrix elements have to be evaluated is four rather than 8 (which is the case for heavy mesons [10]).

In order to obtain the factor $f_{B}^{2} m_{B}$ it is also necessary to determine the normalization of the axial current, $Z_{A}$,

$$
\begin{align*}
f_{B} m_{B} & =\langle 0| A_{0}\left(a^{-1}\right)|B\rangle=Z_{A}\langle 0| A_{0}^{\text {latt }}\left(a^{-1}\right)|B\rangle \\
& =\sqrt{2 m_{B}} Z_{A} Z_{L}, \tag{22}
\end{align*}
$$

where $A_{0}^{\text {latt }}$ is the time component of the axial current defined on the lattice and $Z_{L}$ is obtained from the matrix element determined in numerical simulations At our value of the lattice spacing
$Z_{A}=1-20.0 \frac{\alpha_{s}^{\text {latt }}\left(a^{-1}\right)}{4 \pi} \simeq 0.75$.
In the following section we combine the results for the matrix elements computed on the lattice ( $M_{i}$ and $Z_{L}$ ) with the perturbative coefficients presented in this section ( $h_{i j}$ and $Z_{A}$ ), to obtain the values of $L_{1}\left(m_{b}\right)$ and $L_{2}\left(m_{b}\right)$.

## 3. Lattice computation and results

The non-perturbative strong interaction effects in spectator contributions to inclusive decays are contained in the matrix elements $M_{i}$ in Eqs. (17)-(20). They have been evaluated in a quenched simulation on a $12^{3} \times 24$ lattice at $\beta=5.7$ using the SW improved action [13],

$$
\begin{align*}
S^{\mathrm{SW}}= & S^{\text {gauge }}+S^{\text {Wilson }} \\
& -\frac{i c_{\mathrm{SW}}}{2} \sum_{x, \mu, \nu} \bar{q}(x) F_{\mu \nu}(x) q(x), \tag{24}
\end{align*}
$$

where $S^{\text {gauge }}$ and $S^{\text {Wilson }}$ are the Wilson gauge and quark actions, respectively. We use 20 gauge-field configurations and the light quark propagators are computed using a stochastic inversion based on the exact relation

$$
\begin{align*}
\left(A^{-1}\right)_{i j}= & \frac{1}{Z} \int[d \phi]\left(A_{j k} \phi_{k}\right)^{*} \phi_{i} \\
& \times \exp \left(-\phi_{l}^{*}\left(A^{\dagger} A\right)_{l m} \phi_{m}\right), \tag{25}
\end{align*}
$$

where the $\phi$ are auxiliary bosonic fields, introduced in order to perform the inversion of the matrix $A$, which in our case is the fermionic matrix. To reduce the statistical noise the technique of maximal variance reduction has been used [14]. The use of this stochastic inversion technique makes it possible to compute a light-quark propagator from each point in the half of the lattice with $0<t \leq 12$ (we call this region box I) to each point in the other half where $12<t \leq 24$ (box II). This increases considerably the effective statistics in the computation of the matrix elements.

We have performed the calculations at $c_{\mathrm{SW}}=1.57$, which is the numerical value of $1 / u_{0}^{3}$, with $u_{0}$ being the the average value of a link variable as defined from the trace of the plaquette. The calculation is therefore "tadpole-improved" and hence the perturbative coefficients in Table 1 are the same as those which would be obtained with a tree-level improved action ( $c_{\mathrm{SW}}=1$ ). We have evaluated the matrix elements with two values of the light-quark mass, corresponding to $\kappa_{1}=0.13847$ (for which $m_{\pi} a \simeq 0.74(4)$ ) and $\kappa_{2}=0.14077$ (for which $m_{\pi} a \simeq 0.52(3)$ ) [15]. For this value of $c_{\text {Sw }} \kappa_{\text {crit }} \simeq 0.14351$ [15]. The same
lattice has been used to compute, with satisfactory results, the wave function of a $B$-meson and the effective coupling constant for the decay $B^{*} \rightarrow B+$ $\pi$ (in the Heavy Meson Chiral Lagrangian). This computation is reported in Ref. [16] and the values which we use for $f_{B}^{2} m_{B}$ are extracted from this paper.

The evaluation of the matrix elements requires the computation of two- and three-point correlation functions of the form,

$$
\begin{equation*}
C_{2}\left(t_{x}\right)=\sum_{x}\langle 0| J(x) J^{\dagger}(0)|0\rangle, \tag{26}
\end{equation*}
$$

where we have assumed $t_{x}>0$, and

$$
\begin{align*}
& C_{3}\left(\mathcal{O}, t_{x}, t_{y}\right) \\
& \quad=\sum_{x, y}\langle 0| J(y) \mathcal{O}(0) J^{\dagger}(x)|0\rangle \tag{27}
\end{align*}
$$

where $t_{y}>0>t_{x}$. In Eqs. (26) and (27) $J$ and $J^{\dagger}$ are interpolating operators which can destroy or create the $\Lambda_{b}$ baryon, for which we take
$J_{\gamma}^{\dagger}=\varepsilon_{a b c}\left(\bar{u}_{\alpha}^{a}\left(\gamma^{5} C\right)_{\alpha \beta} \bar{d}_{\beta}^{b}\right) \bar{b}_{\gamma}^{c}$,
where $\bar{u}, \bar{d}, \bar{b}$ are the quark fields. In Eq. (28) $a, b, c$ are colour labels, $\alpha, \beta, \gamma$ are spinor labels and a sum over repeated indices is implied. We define $Z_{\Lambda}$ by
$Z_{\Lambda} u_{\gamma}^{(s)}(\mathbf{0})=\frac{\left\langle\Lambda_{b}, s\right| J^{\dagger}(0)_{\gamma}|0\rangle}{\sqrt{2 m_{\Lambda}}}$,
where $s$ represents the spin state (up or down) of the baryon.

In order to enhance the contribution of the ground state to the correlation functions, it is useful to

Table 1
Matching coefficients for the matrix elements $M_{1}$ and $M_{2}$, renormalised on lattice at an energy scale $a^{-1}=1.10 \mathrm{GeV}$. The values of the integrals $x_{i}$ are reported in the appendices of Ref. [10]

| Coeff. | Expression | Value |
| :--- | :--- | ---: |
| $h_{11}$ | $1+\frac{\alpha}{4 \pi}\left[\frac{10}{3}-\frac{4}{3} x_{1}-\frac{8}{3} x_{2}\right] \simeq 1-21.65 \frac{\alpha}{4 \pi}$ | 0.737 |
| $h_{12}$ | $\frac{\alpha}{4 \pi}\left[-\frac{4}{3} x_{3}\right] \simeq 9.19 \frac{\alpha}{4 \pi}$ | 0.112 |
| $h_{13}$ | $\frac{\alpha}{4 \pi}\left[-\frac{5}{2}-x_{4}-2 x_{5}-x_{7}\right] \simeq 9.29 \frac{\alpha}{4 \pi}$ | 0.113 |
| $h_{14}$ | $\frac{\alpha}{4 \pi}\left[-x_{6}\right] \simeq 6.89 \frac{\alpha}{4 \pi}$ | 0.084 |
| $h_{21}$ | $\frac{\alpha}{4 \pi} \frac{2}{9}\left[-\frac{5}{2}-x_{4}-2 x_{5}-x_{7}\right] \simeq 2.06 \frac{\alpha}{4 \pi}$ | 0.025 |
| $h_{22}$ | $\frac{\alpha}{4 \pi}\left[-\frac{2}{9} x_{6}\right] \simeq 1.53 \frac{\alpha}{4 \pi}$ | 0.019 |
| $h_{23}$ | $1+\frac{\alpha}{4 \pi}\left[\frac{5}{12}-\frac{4}{3} x_{1}+\frac{1}{3} x_{2}-\frac{7}{6} x_{4}+\frac{2}{3} x_{5}-\frac{7}{6} x_{7}\right] \simeq 1-10.82 \frac{\alpha}{4 \pi}$ | 0.869 |
| $h_{24}$ | $\frac{\alpha}{4 \pi}\left[\frac{1}{2} x_{6}\right] \simeq-3.45 \frac{\alpha}{4 \pi}$ | -0.042 |

'smear" the interpolating operators $J$ and $J^{\dagger}$. In this paper we will follow Ref. [14] and adopt the type of smearing known as 'fuzzing'". This technique consists in replacing light quark field $q(x)$, by a 'fuzzed' field

$$
\begin{align*}
q^{F}(x)= & \sum_{i=1,2,3} U_{i}^{F}(x) q(x+\hat{\imath}) \\
& +U_{-i}^{F}(x) q(x-\hat{\imath}) \tag{30}
\end{align*}
$$

where $U_{ \pm i}^{F}(x)$ are defined by the recursive relations

$$
\begin{aligned}
U_{i}^{F}(x)= & \mathscr{P}_{S U(2)}\left[\zeta U_{i}^{F}(x)\right. \\
& +\sum_{j \neq i} U_{j}^{F}(x) U_{i}^{F}(x+\hat{\jmath}) U_{-j}^{F}(x+\hat{\imath}+\hat{\jmath}) \\
& \left.+U_{-j}^{F}(x) U_{i}^{F}(x-\hat{\jmath}) U_{j}^{F}(x+\hat{\imath}-\hat{\jmath})\right]
\end{aligned}
$$

$$
U_{-i}^{F}(x)=\mathscr{P}_{S U(2)}\left[\zeta U_{-i}^{F}(x)\right.
$$

$$
+\sum_{j \neq i} U_{j}^{F}(x) U_{-i}^{F}(x+\hat{\jmath}) U_{-j}^{F}(x-\hat{\imath}+\hat{\jmath})
$$

$$
\begin{equation*}
\left.+U_{-j}^{F}(x) U_{-i}^{F}(x-\hat{\jmath}) U_{j}^{F}(x-\hat{\imath}-\hat{\jmath})\right] \tag{32}
\end{equation*}
$$

starting with initial values $U_{i}^{F}(x)=U_{i}(x)$ and $U_{-i}^{F}(x)=U_{i}^{\dagger}(x-\hat{\imath}) . \mathscr{P}_{S U(2)}$ is a projector on $S U(2)$, implemented as in the Cabibbo-Marinari cooling algorithm, and $\zeta=2.5$ is a constant value. The recursive procedure for $U_{ \pm i}^{F}(x)$ has been applied twice.

We introduce two superscripts on each correlation function, each of which can be either " $F$ ', or " $L$ ', which indicate whether the interpolating operators $J$ and $J^{\dagger}$ are fuzzed or local.

The standard technique to extract hadronic matrix elements of the type $\left\langle\Lambda_{b}\right| \mathcal{O}\left|\Lambda_{b}\right\rangle$ is to look for plateaus in the ratios
$R\left(\mathscr{O}, t_{1}, t_{2}\right)=Z_{\Lambda}^{2} \frac{C_{3}^{F F}\left(\mathscr{O}, t_{1}, t_{2}\right)}{C_{2}^{L F}\left(t_{1}\right) C_{2}^{L F}\left(t_{2}\right)}$.

In our analysis, however, even with the use of fuzzed interpolating operators $J$ and $J^{\dagger}$, we cannot eliminate the effects of excited states from the three-point correlation functions $C_{3}^{F F}\left(\mathscr{O}, t_{1}, t_{2}\right)$ satisfactorily. On the other hand we do find that the ground state dominates the two-point correlation function $C_{2}^{F F}(t)$ for $t>3$, and the masses we obtain in this way agree, within errors, with those found previously on the same lattice [14] using a 3-mass correlated fit for a number of smeared correlators. In order to obtain the matrix elements $M_{i}$ of Eqs. (17)-(20) we therefore need to subtract the effects of the excited states.

We have followed the following procedure to extract the matrix elements $\left\langle\Lambda_{b}\right| \mathscr{O}\left|\Lambda_{b}\right\rangle$ :

- For each value of the light-quark mass we start by fitting the two-point correlation function for $t>3$ with a single exponential

$$
\begin{equation*}
C_{2}^{F F}(t)=\left(Z_{\Lambda_{1}}^{F}\right)^{2} \exp \left(-m_{\Lambda} t\right) \tag{34}
\end{equation*}
$$

thus obtaining the mass of the ground state, $m_{\Lambda}$. Within errors, the masses of the ground state which we obtain are in agreement with those obtained from the same lattice using a more sophisticated fitting procedure in Ref. [14].

- We model the contribution of the excited states by a second exponential and now fit $C_{2}^{F F}(t)$ for $t>1$ by

$$
\begin{equation*}
C_{2}^{F F}(t)=\left(Z_{\Lambda}^{F}\right)^{2} e^{-m_{\Lambda} t}+\left(Z_{\Lambda_{1}}^{F}\right)^{2} e^{-m_{\Lambda_{1}} t} \tag{35}
\end{equation*}
$$

keeping $m_{\Lambda}$ and $Z_{\Lambda}^{F}$ fixed at the values obtained from the single exponential fit above. We find that $C_{2}^{F F}$ is well represented by the two exponentials.

- For each operator $\mathscr{O}$ we then fit the three-point correlation function $C_{3}^{F F}\left(\mathscr{O}, t_{1}, t_{2}\right)$ to

$$
\begin{align*}
C_{3}^{F F} & \left(\mathscr{O}, t_{1}, t_{2}\right) \\
= & \frac{\left\langle\Lambda_{b}\right| \mathscr{O}\left|\Lambda_{b}\right\rangle}{2 m_{\Lambda}}\left(Z_{\Lambda}^{F}\right)^{2} e^{-m_{\Lambda}\left(\left|t_{1}\right|+t_{2}\right)} \\
& +C\left[e^{-m_{\Lambda}\left|t_{1}\right|-m_{\Lambda_{1}} t_{2}}+e^{-m_{\Lambda}\left|t_{1}\right|-m_{\Lambda_{1}} t_{2}}\right] \tag{36}
\end{align*}
$$

obtaining values for the two unknown parameters $\left\langle\Lambda_{b}\right| \mathscr{O}\left|\Lambda_{b}\right\rangle$ and the constant $C$, which encodes the contribution from excited states.
This procedure has been repeated for each of the 4 relevant operators, and for the linear combinations corresponding to $L_{1}$ and $L_{2}$ on 40 jackknife samples to extract the statistical errors.

In order to control the contributions from the excited states more effectively it will be necessary to carry out a simulation with considerably improved statistics. It is, however, possible to check the consistency of our approach a posteriori. We subtract the contributions from the excited states obtained above from the two- and three-point correlation functions, and look for plateaus in the ratios:
$R\left(\mathscr{O}, t_{1}, t_{2}\right)=Z_{\Lambda}^{2} \frac{\tilde{C}_{3}^{F F}\left(\mathscr{O}, t_{1}, t_{2}\right)}{\tilde{C}_{2}^{L F}\left(t_{1}\right) \tilde{C}_{2}^{L F}\left(t_{2}\right)}$,
where the tilde indicates that contribution from excited states has been subtracted from the correlation function. The subtracted two-point correlation function is reported in Fig. 1. Fig. 2 shows the plateaus for the ratios $R$ corresponding to the operators in $L_{1}$ and $L_{2}$ (with the appropriate normalization factor $\left.\frac{8}{f_{B}^{2} m_{B}}\right)$. The plateaus in Fig. 2 give us confidence in our treatment of the subtraction of the contribution of the excited states. The results for the matrix elements obtained from Eqs. (36) and (37) agree to within $1 \%$.

Our results for the matrix elements at each of the two values of $\kappa$ are reported in Table 2. Combining them with Eq. (2) we obtain

$$
\frac{\tau\left(\Lambda_{b}\right)}{\tau\left(B^{0}\right)}= \begin{cases}0.91(1) & \text { for } a m_{\pi}=0.52(5)  \tag{38}\\ 0.93(1) & \text { for } a m_{\pi}=0.74(4)\end{cases}
$$

From Eq. (38) we see that, although they are of $O\left(1 / m_{b}^{3}\right)$, spectator effects are indeed significant (compare Eqs. (38) and (2)). Estimates of the parameter $r$ defined in Eq. (4), using the non-relativistic quark model or the bag model $[17,18]$ or QCD Sum Rules [19] are typically in the range $0.1-0.5$. On the


Fig. 1. Plot of $C_{2}^{F F}$ after the subtraction for the contribution of excited states.


Fig. 2. Plot of the matrix elements $L_{1}\left(a^{-1}\right)$ and $L_{2}\left(a^{-1}\right)$ computed after the subtraction of excited states.
other hand, Rosner has estimated $r$ from the spin splitting between $\Sigma_{Q}$ and $\Sigma_{Q}^{*}$ baryons ( $\mathrm{Q}=\mathrm{c}, \mathrm{b}$ ) and finds $r \simeq 1$ (2) from charmed (beauty) baryons [20]. A recent reanalysis of this problem using QCD Sum Rules in which more condensates are introduced, finds a range of possible values for the ratio of lifetimes (including ones close to the experimental value in Eq. (1)), depending on the (unknown) values of various condensates [21]. At the two values of the light quark mass at which we do our computations we find $r \simeq 1.2 \pm 0.2$, which is at the high end of expectations. The large values which we find for $r$ and consequently the significant effect on the prediction for the lifetime ratios, make it important to improve the precision of the lattice simulations.

In quark models and related pictures, the parameter $\tilde{B}=1$. In our simulations we find larger values, $\tilde{B}=1.9 \pm 0.2(1.3 \pm 0.2)$ at $\kappa_{1}\left(\kappa_{2}\right)$.

## 4. Conclusions

In this paper we have evaluated the matrix elements which contain the non-pertubative QCD ef-

Table 2
Lattice results for the matrix elements computed on lattice, $M_{i}$, the combined matrix elements at two different scales, $L_{i}$, and the physical ratio of lifetimes

| Expression | $\kappa_{1}$ | $\kappa_{2}$ |
| :--- | :---: | :---: |
| $Z_{A}^{2} Z_{L}^{2}=f_{B}^{2} m_{B} / 2$ | $0.33(1)$ | $0.33(1)$ |
| $M_{1}$ | $-0.026(3)$ | $-0.019(3)$ |
| $M_{2}$ | $0.045(4)$ | $0.039(5)$ |
| $M_{3}$ | $0.018(2)$ | $0.013(2)$ |
| $M_{4}$ | $-0.040(4)$ | $-0.031(4)$ |
| $L_{1}\left(a^{-1}\right)$ | $-0.18(2)$ | $-0.13(3)$ |
| $L_{2}\left(a^{-1}\right)$ | $0.21(2)$ | $0.16(2)$ |
| $L_{1}\left(m_{b}\right)$ | $-0.31(3)$ | $-0.22(4)$ |
| $L_{2}\left(m_{b}\right)$ | $0.23(2)$ | $0.17(2)$ |

fects in the spectator contribution to inclusive decays of the $\Lambda_{b}$ baryon. Our principal results (in the $\overline{\mathrm{MS}}$ scheme) are presented in Table 2. The results indicate that spectator effects are important, accounting for a significant fraction of the discrepancy between the theoretical prediction for $\tau\left(\Lambda_{b}\right) / \tau\left(B_{d}\right)$ in Eq. (2) and the experimental result in Eq. (1). It also appears that not all of the discrepancy can be accounted for by spectator effects.

The calculation described in this paper is the first evaluation, using lattice simulations, of the matrix elements in Eq. (8) and Eq. (9) between $\left|\Lambda_{b}\right\rangle$ states. Having established that spectator effects are significant, it is now necessary to improve the precision, both statistical and systematic. This requires a highstatistics simulation at a smaller value of the lattice spacing (to decrease the errors due to discretisation) and with more values of the light quark mass (to enable a reliable extrapolation to the chiral limit).

In this study we have used static $b$-quarks. It would also be valuable, as a control of the systematic errors, to repeat the calculation with propagating $b$-quarks, for which these uncertainties are different.

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[^0]:    ${ }^{1}$ Eq. (5) can be derived from Eq. (39) of Ref. [9] by the substitution
    $\tilde{B}=-6 L_{1}$,
    $r=-2 \frac{L_{2}}{L_{1}}-\frac{1}{3}$.
    ${ }^{2}$ The coefficients of two of the matrix elements for $B$-mesons are so small that we don't include them in Eq. (5).

